Taking Propositions Apart and Telling Them Apart

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> *While some entailments are like beams contained in a house, others are like plants contained in their seeds*. — Frege[1]

A logic is a theory of propositional *consequence* or *entailment*. Ken Gemes (1994) argued that in addition to a logic, propositions also have a *mereology* (cf. also Angell 1977):

> **Propositional Mereology**. Propositions can contain other, weaker propositions as *parts*. This fundamental containment relation is distinct from consequence.

More recently, a slew of philosophical logicians have taken up and developed Gemes' vision (Humberstone 2000; Yablo 2014; Fine 2013, 2017; Hawke 2016; Goodman 2019; Berto 2022). We can think of containment/parthood as an especially *direct* or *immediate* kind of entailment. The paradigm illustration of the distinction is ***conjunction vs. disjunction***:

1) Ellen likes cauliflower and James likes broccoli
 contains
 James likes broccoli

2) Jill took her aspirin this morning.
 does not contain
 Either Jill took her aspirin this morning or she has emigrated to Barbados.

Just as with entailment, the concept of parthood extends to other types (properties, relations, operators…). E.g. *studying language* may be part of *being a linguist*.

Two main aims today:

- Argue that linguists have use for a notion of parthood — ***Join the Parthood Party!***
- Show how questions about the nature of parthood link up with questions on the nature/individuation of propositions, giving us a better empirical hold on the latter.

Application 1: Speech Reports

We can truthfully report the speaker as having said some but not all of what is entailed by the assertions they made (Brasoveanu and Farkas 2007, Sæbø 2013, Abreu Zavaleta 2019):

3) Anne: "Richard is nasty, brutish and short."
 $\rightsquigarrow$ a. Anne said that *Richard is nasty*.
 $\rightsquigarrow$ b. Anne said that *Richard is short and brutish*.
 $\not\rightsquigarrow$ c. Anne said that *Richard is either nasty or incompetent*.
 $\not\rightsquigarrow$ d. Anne said that *all bachelors are unmarried*.

4) Teacher reads off some test results from a grade book: "Ahmed got a B. Blake got…"
 $\rightsquigarrow$ a. The teacher said that *Jill and Zeynep both got an A on the test*.
 $\not\rightsquigarrow$ b. The teacher said that *at least seven students got a B on the test*.

[1] Paraphrase. See *Foundations of Arithmetic*, §88.

Hypothesis. ***A speech report is true just in case the prejacent expresses a proposition that is part of what the speaker said***.

- So (1a-b) are true because they report *part of what Anne said*, and (1c-d) are false because they report entailments of what Anne said that were not part of the proposition she expressed.
- If a speaker makes multiple assertions in a row, as in (4), it is natural to suppose they have said the *conjunction* of the propositions they have asserted; the prejacent of (4a) is part of that conjunction but the prejacent of (4b) is not.

Two Accounts of Parthood

Subject Matter Parthood (Hawke 2016, Berto 2018, Hoek 2025)

- A proposition p is an ordered pair of a *subject matter* or a *question* $\sigma(p)$ and a set of worlds $\tau(p)$ representing subject-responsive truth conditions (a way things could be w.r.t. the subject matter in question).
  - $p \wedge q = \langle\, \sigma(p) \oplus \sigma(q),\ \ \tau(p) \cap \tau(q) \,\rangle$
  - $p \vee q = \langle\, \sigma(p) \oplus \sigma(q),\ \ \tau(p) \cup \tau(q) \,\rangle$
  - On one development of this view, subject matters are partitions of logical space, and '$\oplus$' is coarsest common refinement; $\tau(p)$ is a union of $\sigma(p)$-partition cells.
- Proposition p is ***part of*** q iff (i) q entails p and (ii) the subject matter of q encompasses the subject matter of p.

  (Or: (i) $\tau(q) \subseteq \tau(p)$ and (ii) $\sigma(p) \oplus \sigma(q) = \sigma(q)$.)

Truthmaker Parthood (Yablo 2014, Fine 2018):

- A proposition is a set of *states* or truthmakers, each representing a *way* the proposition can be true.
  - $p \wedge q = \{\, P \sqcup Q : P \in p,\ Q \in q \,\}$
  - $p \vee q = p \cup q$
- Proposition p is ***part of*** q iff every truthmaker for p is part of truthmaker for q, and every truthmaker for q contains a truthmaker for p.

  (Or: $\forall P \in p\ \exists Q \in q$ such that $P \sqsubseteq Q$, and $\forall Q \in q$, $\exists P \in p$ such that $P \sqsubseteq Q$)

Application 2: Attitude Closure

Claims about what somebody knows or believes apparently entail claims about what other propositions they know or believe:

5) a. Ryan knows [thinks] that it's 8.30pm.
   ⇝ b. Ryan knows [thinks] that *it's not 4.30pm*.
   ⇝ c. Ryan knows [thinks] that *it's evening*.
   ⇝ d. Ryan knows [thinks] that *it's not yet 9pm*.

This naturally invites the generalisation that knowledge and belief are closed under entailment, as indeed they are on traditional semantic accounts of 'know' and 'believe' (Hintikka 1962; Heim and Kratzer 1990). But that generalisation is too strong:

6) a. Emma thinks the car keys are nowhere in the house.
$\not\rightsquigarrow$ b. Emma thinks *the car keys aren't in the second drawer of the little cabinet by the door*.

7) a. Anne knows she has 13 full egg cartons in the bag
$\not\rightsquigarrow$ b. Anne knows she has *156 eggs in the bag*.

8) a. King William believes he can avoid war with France. (Stalnaker)
$\not\rightsquigarrow$ b. King William believes *he can avoid nuclear war with France*.

9) a. Frank thinks he turned off the stove. (Kripke)
$\not\rightsquigarrow$ b. Frank thinks that all evidence to the contrary is misleading.

Entailments often seem less *justified* than the proposition that entails them:

10) a. Zeynep knows she will teach logic next year.
$\not\rightsquigarrow$ b. Zeynep knows she will not be killed by lightning next month.

11) a. Fred knows that that is a zebra.
$\not\rightsquigarrow$ b. Fred knows that that is not a cleverly disguised mule.

Yablo: "*[The unknown entailment] Q raises additional issues, not contemplated in P. It is Q's claims about these additional issues that make it harder to know… it has jagged edges, newly exposed flanks… you pick the metaphor. An irregular, disunified region is not as defensible as a compact region with smooth boundaries.*" (2014, 117-8)

Hypothesis. ***Cognitive attitudes like knowledge and belief are closed under <u>parthood</u>, not entailment.***

- You may not know every entailment of what you know, but you do know every *part* of what you know (Hawke 2016, Yablo 2017).
- Likewise for belief: you needn't necessarily believe every entailment of what you believe, but you do believe every *part* of what you believe (Hoek 2025).

Application 3: Partial Truth

A proposition is ***partly true*** if it has a (substantive) true part (Yablo 2014, Fine 2025).

Do collaborative speakers always aim to say the *whole truth*?

12) I haven't seen Ed in a hundred years.
True part: I haven't seen him in the past three weeks.

13) Everybody loves Beyonce.
True part: Abby and Johnny and Katie and Jacky and… all love Beyonce.

14) The fridge is empty.
True part: There is no food in the fridge. [It is not true that there are no crumbs/dust particles/air in the fridge.]

Hypothesis. ***Sincere collaborative speakers do not always aim to speak the truth. Sometimes, as when using hyperbole, they merely aim to say something that is partly (or largely) true***.

(*Side note*: this would help explain why it is often assumed — incorrectly, I think — that hyperbole is a species of loose talk.)

Partial truth without hyperbole. Partial truth does not in general make claims assertable:

15) # Paris is the capital of France and Stockholm is the capital of Finland.
True part: Paris is the capital of France.

16) # Mary has three cars and a canary..
True part: Mary has one car.

The partial truth condition was only. What are others? Candidates:

- No known (simpler) alternative leaves out a falsehood while preserving the true part.
- Falsehood must be *evident*. (Kao, Wu Bergen and Goodman 2014)

Hyperintensional hyperbole. Consider an observation from Diego Feinmann (2024):

17) This exercise is impossible to solve!
$\rightsquigarrow$ This exercise is very difficult to solve.

18) This exercise has no solution!
$\not\rightsquigarrow$ This exercise is very difficult to solve.

Explanation: Assuming the exercise is in fact very difficult but not impossible, (15) but not (16) is partly true. The exercise's difficulty goes some way towards making it true that the exercise is impossible to solve, as a large obstacle goes some way towards an insuperable obstacle. But it does not go any way towards making it true that the exercise has no solution.

Or: (15) addresses the subject matter *how difficult the exercise is*, and entails substantive truths about that question; (16) addresses the subject matter *whether the exercise has a solution* and entails no substantive truths about that. Another telling contrast from Feinmann:

19) Stephen King has won every literary prize.
$\rightsquigarrow$ Stephen King has won many prizes.

20) # Stephen King has won the Nobel Prize.
$\not\rightsquigarrow$ Stephen King has won a prestigious prize.

Parthood and Hyperintensionality

A theory of parthood requires a hyperintensional setting, since the following three claims are inconsistent:

Intensionality. If two propositions p and q are mutually entailing, then $p = q$.
Conjuncts. Any conjunction $(p \wedge q)$ contains each of its conjuncts p and q as a part.
Non-Trivial Parts. It is not the case that p contains q whenever p entails q.

Suppose p entails q but does not contain q. By *Conjuncts*, $(p \wedge q)$ does contain q so $p \neq (p \wedge q)$. But p and $(p \wedge q)$ are mutually entailing. So *Intentionality* must fails if parthood is to succeed!

Desideratum for a hyperintensional theory of propositions: it should support a notion of parthood that could do (some of) the theoretical work suggested by our applications.

A Recipe for Parts

In classical mereology, one simple way to characterise the *fusion* of two things is as their *least upper bound*: the smallest entity that contains both (e.g. Bostock 1979, van Benthem 1983; see Hovda 2009 for alternatives):

21) $x \oplus y = z \;\; \leftrightarrow_{\mathrm{df}} \; \forall w\, ((w \geq x \wedge w \geq y) \rightarrow w \geq z)$

Here '⊕' stands for fusion and '≥' for containment. Alternatively, one may start with a primitive notion of least upper bound, and then define parthood thus:

22) $x \geq y \; \leftrightarrow_{\mathrm{df}} \; ((x \oplus y) \; = x)$

We encounter the same structure in the standard logic of plurals (Link 1983, Landman 1989):

23) $xx + yy = zz \;\; \leftrightarrow \;\; \forall ww\, ((ww \geq xx \wedge ww \geq yy) \rightarrow ww \geq zz)$

24) $xx \geq yy \; \leftrightarrow \; ((xx \oplus yy) \; = xx)$

In most languages, "and" is used to fuse pluralities:

25) Amy Winehouse and Jimmy Hendrix and the Beatles

26) Those knives over there, the forks from the attic and Emma's spoon

What else is "and" used for? …

Proposal. To conjoin propositions! This suggests propositional *fusion* is in fact familiar:

proposition fusion = proposition conjunction

If that is right, we can use the well known concept of conjunction, together with the concept of propositional identity, to analyse the more exotic concept of propositional parthood. In the language of higher-order logic:

27) $\forall_t pq.\;\; p \geq_t q \;\; \leftrightarrow \;\; ((p \wedge q) \; =_t p)$

28) $\lambda pq.\, p \geq_t q \;\; =_{t \rightarrow t \rightarrow t} \; \lambda pq.((p \wedge q) \; =_t p)$

Here (27) says that the parthood relation is and the RHS are *coextensive*: they relate the same propositions. (28) makes the stronger claim that the parthood relation is *identical* to the propositional relation expressed on the RHS. In what follows, we just need (27).

Applying (27) to some examples:

- *Stephen is skinny* is part of *Sam and Stephen are skinny* just in case the proposition that *Sam and Stephen are skinny and Stephen is Skinny* is identical to the proposition that *Obama is tall and handsome*.
- *Mira studies language* is part of *Mira is a linguist* just in case *for Mira to be a linguist and study language* **just is** *for Mira to be a linguist*. (Rayo 2013)

Other types. Conjunction can be used to combine entities of many types: properties ("free and brave"), relations ("married and in love"), operators ("probably and happily")… This allows us to derive corresponding notions of parthood for any functional type τ:

29) $\forall_\tau PQ.\;\; P \geq_\tau Q \; \leftrightarrow \; ((P \wedge_\tau Q) \; =_\tau P)$

30) $\lambda PQ.\, P \geq_\tau Q \; =_{\tau \rightarrow \tau \rightarrow t} \;\; \lambda PQ.((P \wedge_\tau Q) \; =_\tau \; P)$

Applying (29) to some examples:

- *being unmarried* is part of *being a bachelor* just in case the property of *being a bachelor* and the property of *being an unmarried bachelor* are identical.
- *possibly* is part of *probably* just in case *possibly and probably* is the same as *probably*.

Testing Propositions by Taking Them Apart

We want a concept of parthood that lives up to its potential, helping us out with *speech reports*, *attitude reports*, *hyperbole* and an open-ended list of other semantic and pragmatic problems.

Given our proposed recipe for parthood, these desiderata carry over to a discriminating test for any theory of propositions: given (29) identity criteria for propositions and a semantics for conjunction jointly determine a precise characterisation of parthood.

At a minimum, we need this characterisation to satisfy following:

- *Conjuncts.* Any conjunction $(p \wedge q)$ contains each of its conjuncts p and q as a part.
- *Disjuncts.* It is not the case that a disjunct p always contains the disjunction $(p \vee q)$.
- *Reflexivity.* Propositions contain themselves as parts.
- *Transitivity.* If p is part of q, and q is part of r, then p is part of r.

We can use these criteria to 'test drive' various theories of propositions.

Some theories unambiguously fail the test:

- ***Intensional theories*** of propositions individuate propositions truth-conditionally. On any such theory, $(p \wedge q) =_t p$ whenever p entails q. So by the recipe, $p \geq_t q$ whenever p entails q. Hence these theories fail the test: they can't satisfy *Non-Triviality* or *Disjuncts*.
- ***Structured theories*** of propositions, such as Russellian propositions or those of Bacon (2023), individuate propositions partly by their syntactic structure. On any such theory, $(p \wedge q) \neq_t p$ no matter what p and q are, and so no proposition is part of another.[2]

For other approaches to propositional accounts, the tests substantively constrain the development of the theory:

- ***Subject matter theories*** pass the test provided they adopt a conception of subject matters according to which they 'absorb' smaller subject matters.
  - *Absorption*. $\sigma_1 \oplus \sigma_2 = \sigma_1$ whenever σ_1 encompasses σ_2
  - E.g. partitional subject matters satisfy this constraint, as do questions from standard inquisitive semantics.
- ***Truthmaker theories*** of propositions do not generally pass the test:
  - *Conjuncts* may fail, since it is not guaranteed that e.g. $(p \wedge q) \wedge p = (p \wedge q)$
  - Suppose $p = \{P_1, P_2\}$ and $q = \{Q\}$. Then $(p \wedge q) \wedge p$ contains $P_1 \sqcup Q \sqcup P_2$ given the

[2] An advocate of structured theories may respond by adopting a different definition of parthood, namely the following: $p \geq q$ iff $p = \bigwedge_i r_i$ and $r_j = q$. This definition makes *Conjuncts* and *Reflexivity* automatic. However, it gives up on a key part of the vision by relinquishing the idea of conjunction as fusion/least upper bound (e.g. $p \wedge p$ is not the least upper bound of p and p). Moreover, it is clear that this notion of parthood is not at all suitable for any of the interesting applications discussed.

semantics of conjunction. But it is not guaranteed that $(p \wedge q)$ contains this state.

- Likewise, *Reflexivity* fails where $(p \wedge p)$ contains $P_1 \sqcup P_2$, but p does not.
- Both problems may be fixed by requiring that propositions be *regular*. (Fine 2017)
  - This means they have to be *closed*: if P_1, $P_2 \in p$, then $P_1 \sqcup P_2 \in p$. And also *convex*: if P_1, $P_2 \in p$ and $P_1 \sqsubseteq S \sqsubseteq P_2$, then $S \in p$.

- The propositions from standard-issue ***Inquisitive Semantics*** (Ciardelli, Groenendijk and Roelofsen 2019) validate *Non-Triviality*, but violate *Disjuncts*.
  - That is, the theory validates $p \wedge (p \vee q) =_t p$.
  - However, if we adopt ideas from *Radical Inquisitive Semantics* (Groenendijk and Roelofsen 2010) we can get a notion of parthood that does satisfy the constraints.
    - Radical Inquisitive Semantics employs *bilateral* propositions.
    - This makes them inquisitive in both a forward- and a backward-looking way: they capture the question *addressed* as well as the question *remaining*.
    - This brings Inquisitive Semantics closer to the subject matter approach.

Using these basic properties to constrain hyperintensional theories of propositions is just the beginning: if we get a handle on the empirical manifestations of parthood, it can help us get a grip on how the hyperintensional content of sentences is determined.

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