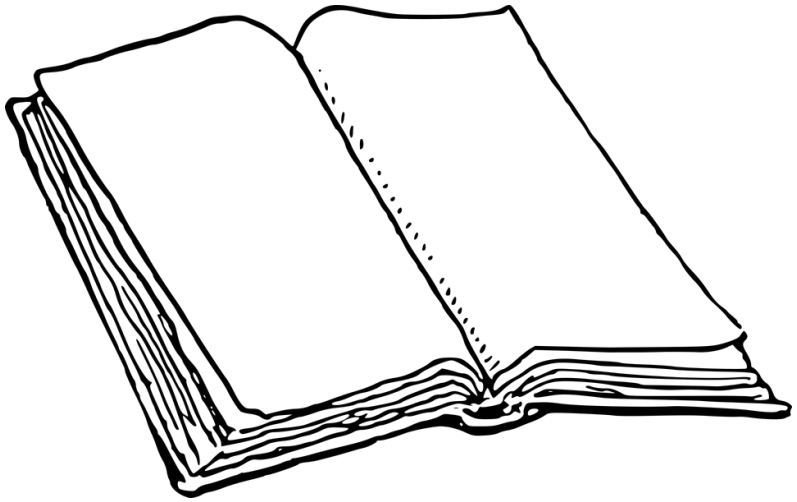
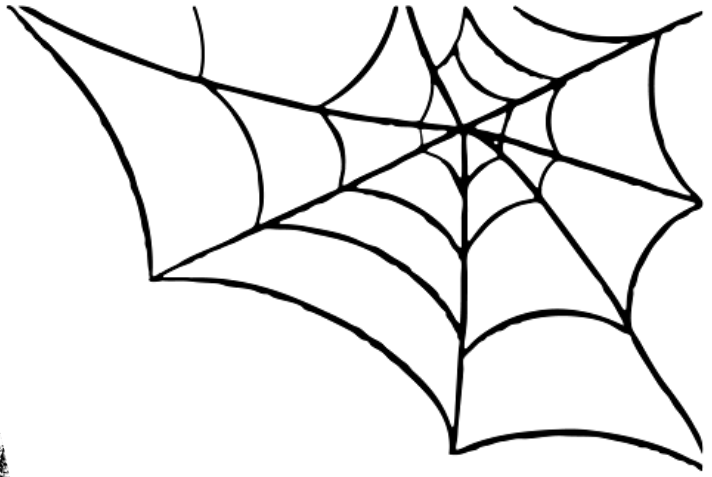




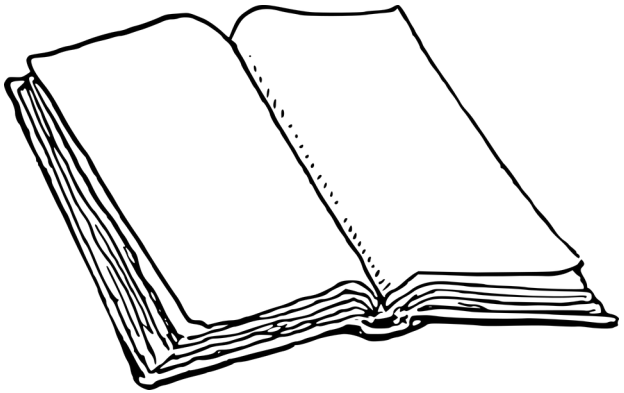
The Atlas or the Web?

daniel.hoek@vt.edu

LEMM Talk Series, University of Toronto, 5 November 2021

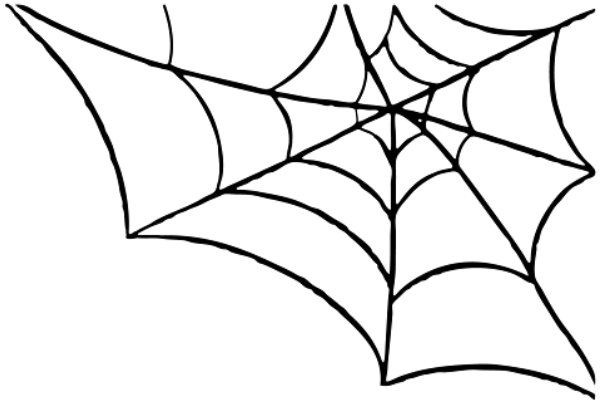
Rescher & Brandom 1980
David Lewis 1982
Levesque 1984
Stalnaker 1984, 1991, 1999
Fagin & Halpern 1988
Egan 2008
Yalcin 2011, 2018, 2021
Greco 2015
Elga & Rayo 2021a, b
Borgoni et al. (eds.) 2021

Gemes 1994, 1997
Fine 2013, 2016, 2017
Yablo 2014, 2017
Hawke 2016
Hoek 2019, 2020; Hoek & Bradley ms.



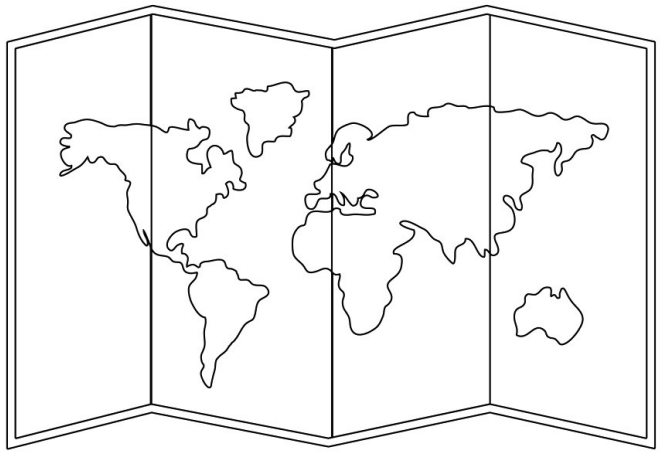
“The Atlas”

(Fragmentation Theories /
Fragmented Decision Theory)



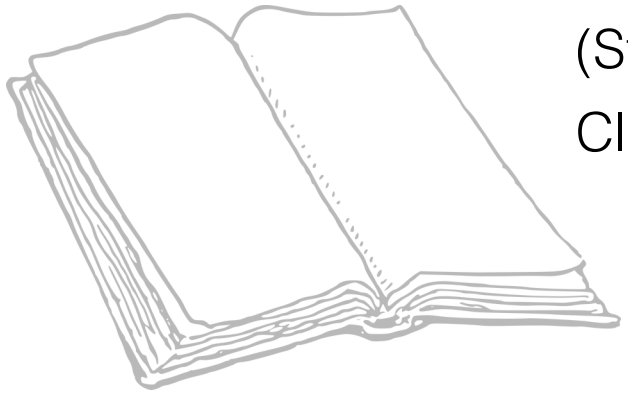
“The Web”

(The Web of Questions Model /
Inquisitive Decision Theory)



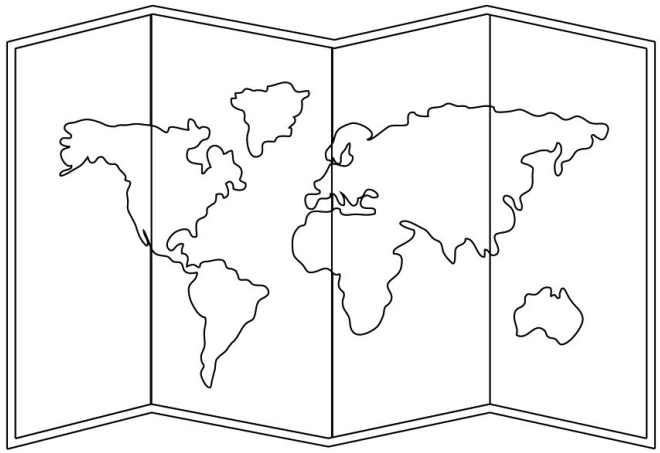
“The Map”

(Standard Doxastic Logic /
Classical Decision Theory)

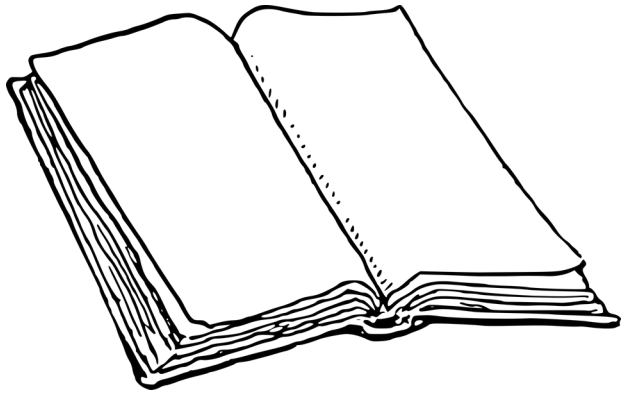


“The Atlas”

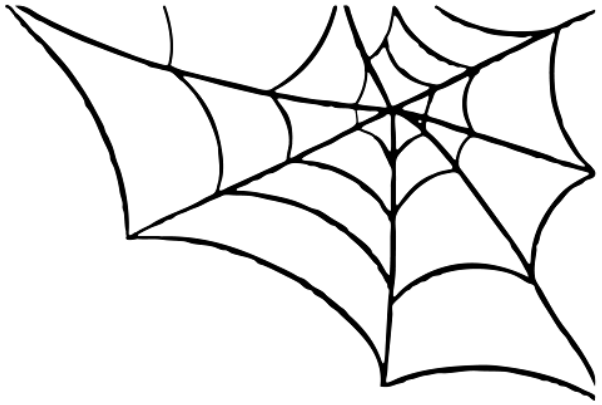
“The Web”



“The Map”

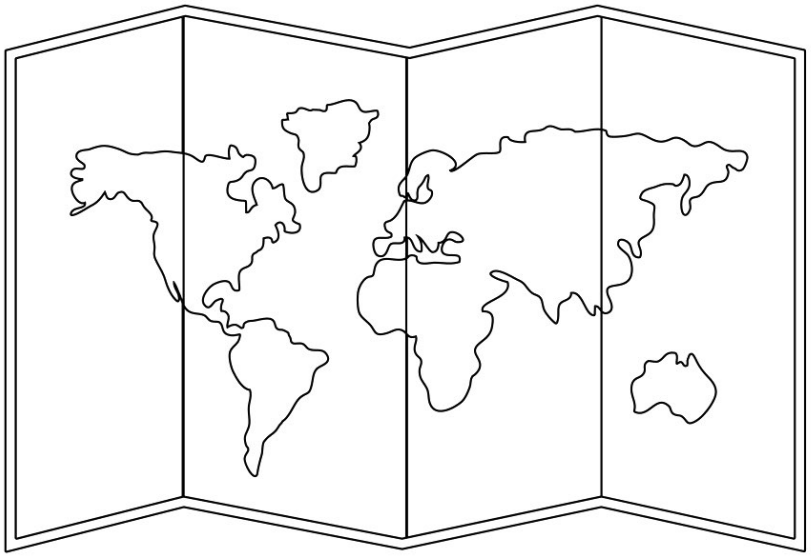


“The Atlas”



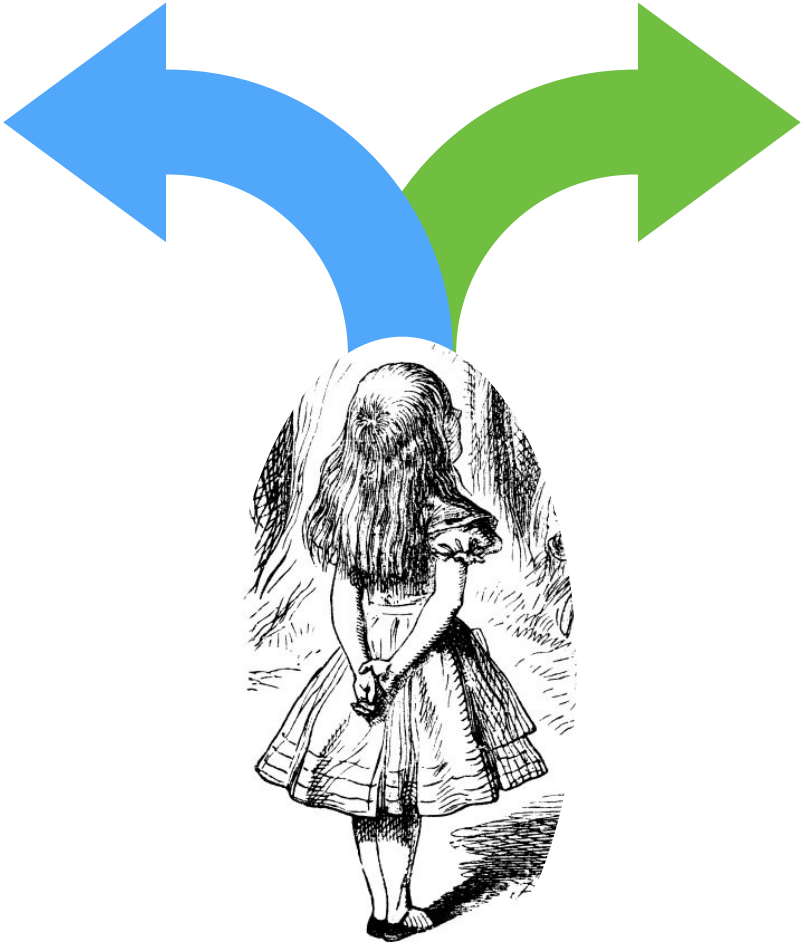
“The Web”

The Map



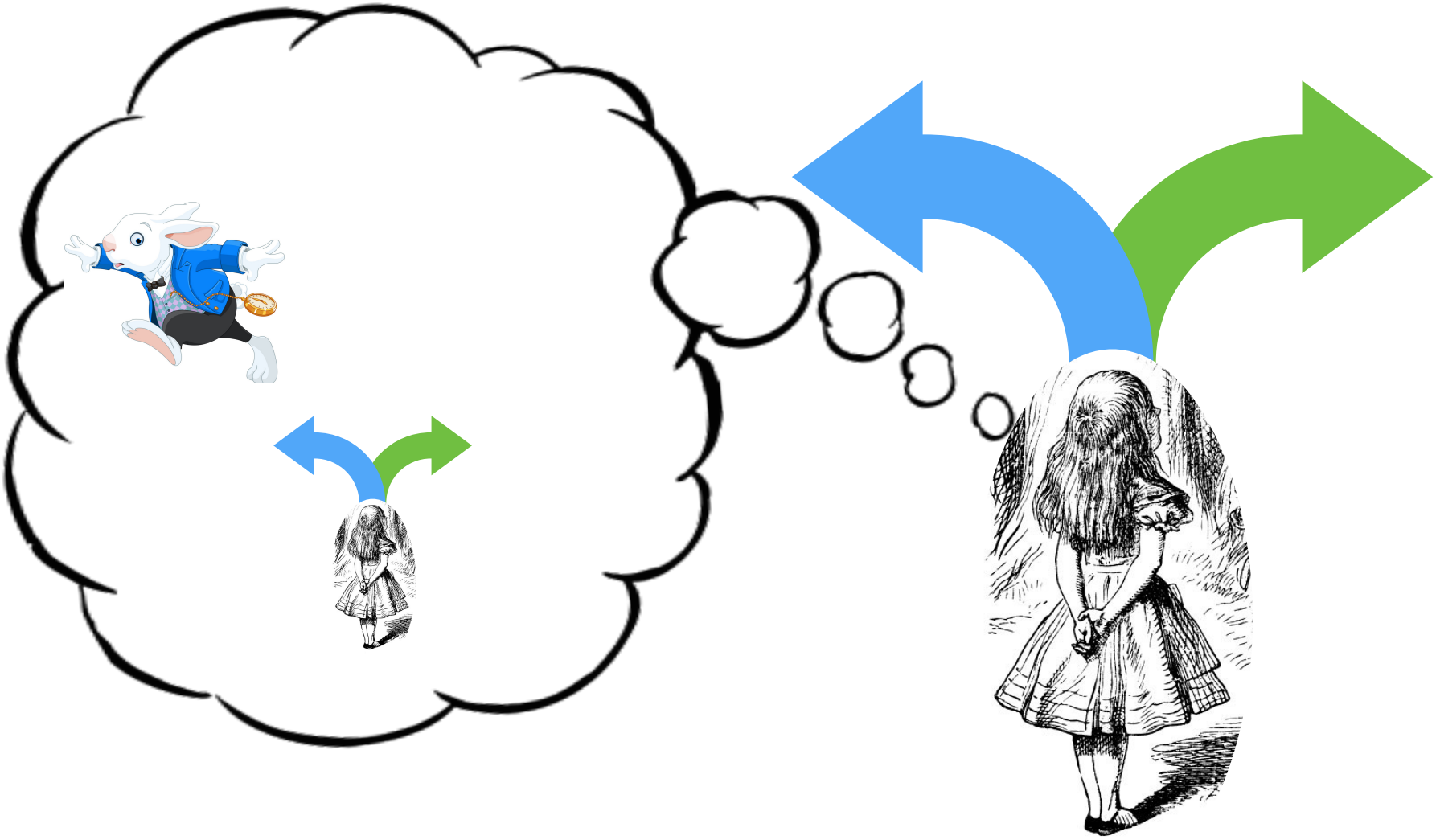
“Beliefs are the map by which we steer” (Ramsey)

The Map



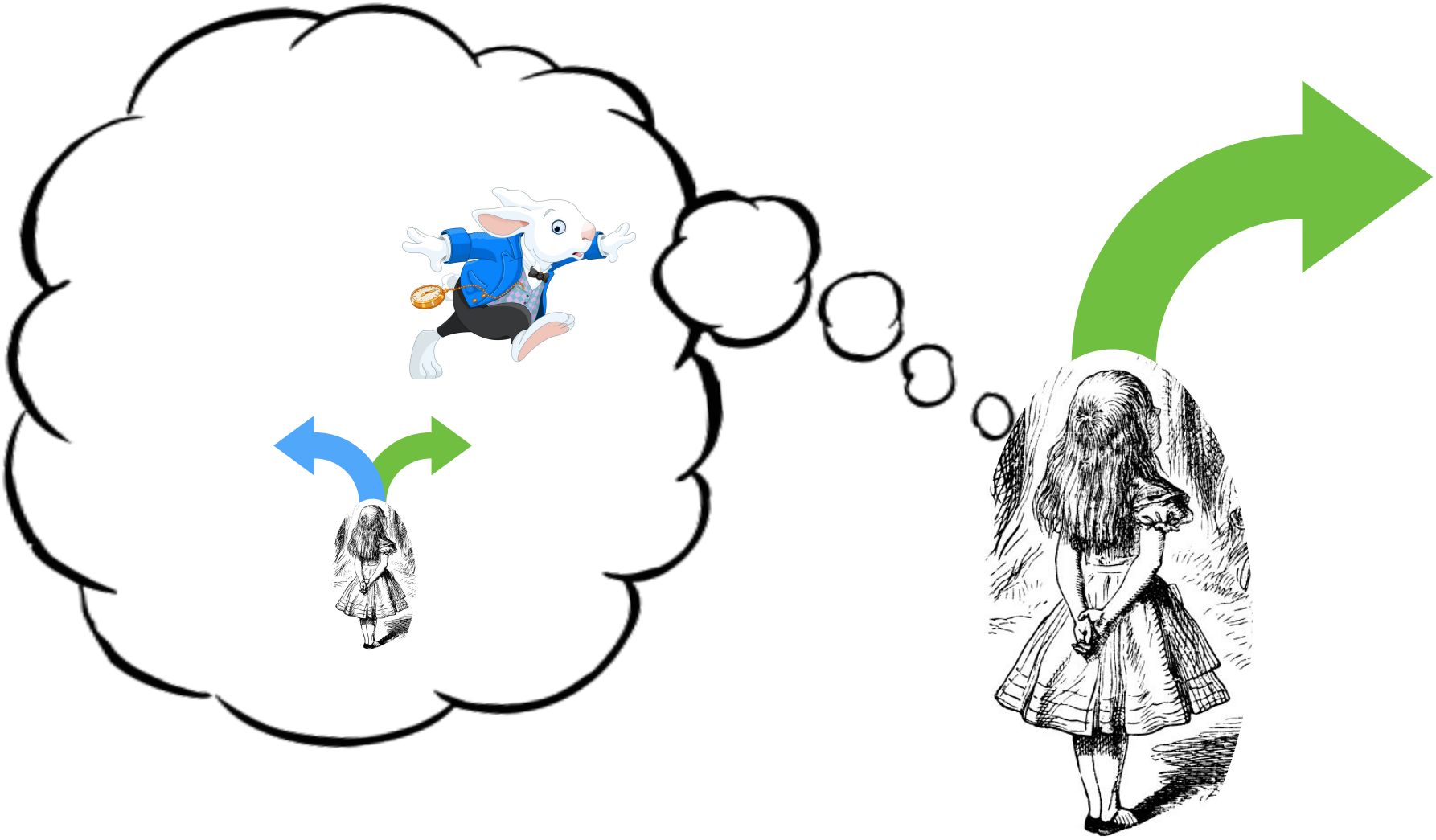
"Beliefs are the map by which we steer" (Ramsey)

The Map



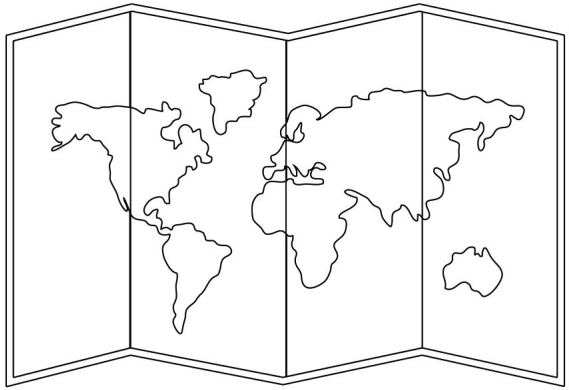
"Beliefs are the map by which we steer" (Ramsey)

The Map



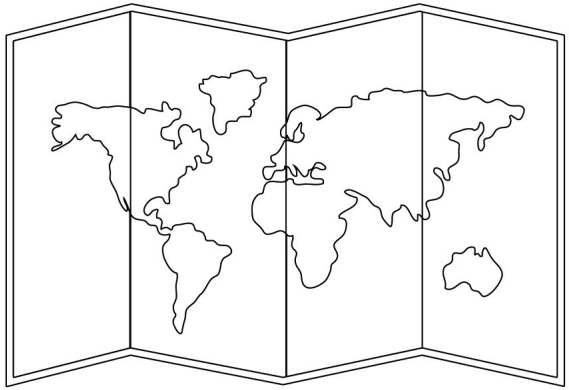
"Beliefs are the map by which we steer" (Ramsey)

The Map



- All of our individual beliefs are different aspects of a monolithic, global world view of the world.
- We act in ways that would tend to promote our aims if that world view were correct.

The Map

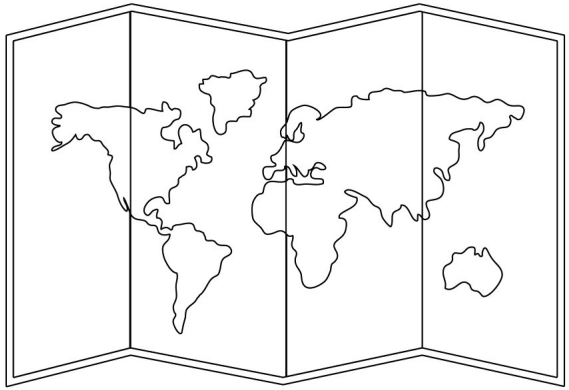


- A *classical belief state* is a non-empty set **D** of possible worlds.

- An agent in state **D** believes *p* just in case *p* is true at all worlds in **D**.

- *Classical Decision Rule*: An agent in state **D** performs the action that, at worlds in **D**, would be most beneficial.

The Map

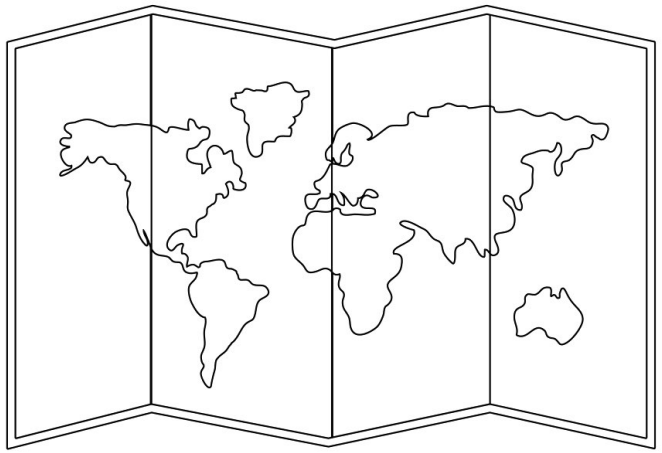


Let p_1, p_2 … p_n be inconsistent propositions. Then

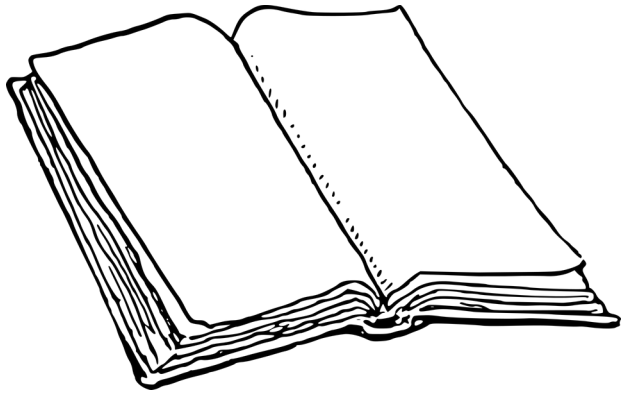
- *Consistency*: $\mathrm{B}p_1, \mathrm{B}p_2 \ldots \mathrm{B}p_{n-1} \models \neg\mathrm{B}p_n$
- *Multi-Premise Closure*: $\mathrm{B}p_1, \mathrm{B}p_2 \ldots \mathrm{B}p_{n-1} \models \mathrm{B}\neg p_n$

Virtues of the Map

- ✓ Explains how *belief predicts behaviour*
- ✓ Accounts for *mind reading*
- ✓ Yields a simple theory of *learning* and the *dynamics of belief*
- ✓ Fits into a *functionalist view of the mind* (esp. in connection with the representation theorems)



“The Map”



“The Atlas”

“The Web”

(Fragmentation Theories /
Fragmented Decision Theory)

Vices of the Map

- ✗ Can't account for *inconsistent beliefs / behaviour*
- ✗ Can't account for *failures of deductive closure*
- ✗ Can't account for the distinction between *recognition and recall*

Recognition vs. Recall

"Alice's phone number is 529 6300"

Inconsistent Beliefs

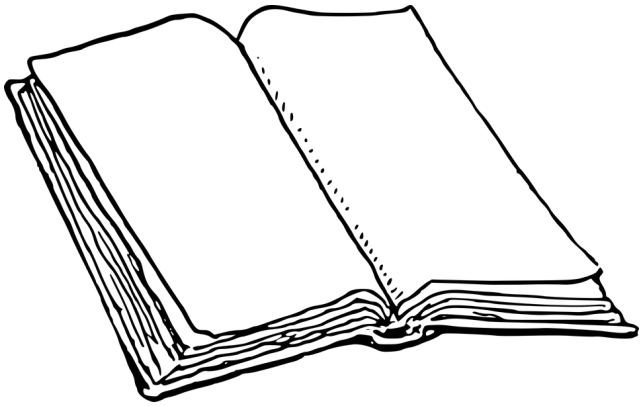
"I used to think that Nassau Street ran roughly east-west; that the railroad nearby ran roughly north-south; and that the two were roughly parallel… So each sentence in an inconsistent triple was true according to my beliefs, but not everything was true according to my beliefs…"

Inconsistent Beliefs

"Now, what about the blatantly inconsistent conjunction of the three sentences? I say that it was not true according to my beliefs. My system of beliefs was broken into (overlapping) fragments. Different fragments came into action in different situations, and the whole system of beliefs never manifested itself all at once"

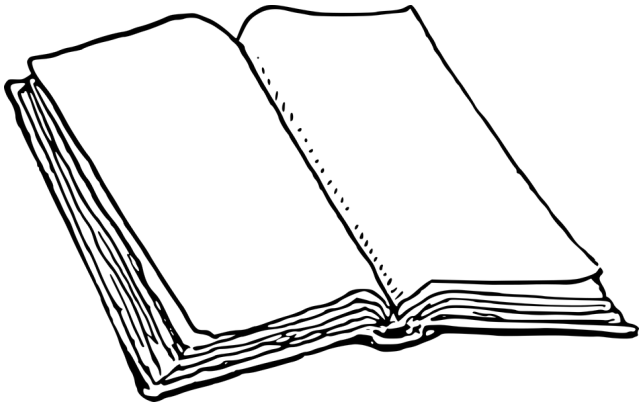
(Lewis 1982)

The Atlas



"Rather than having a single system of beliefs that guides all of our behavior all of the time, we have a number of distinct, compartmentalized systems of belief, different ones of which drive different aspects of our behavior in different contexts" (Egan 2008)

The Atlas

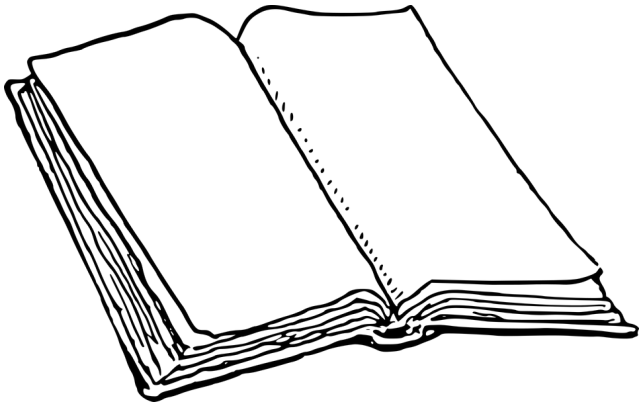


- *Divided Mind*: Agents can simultaneously occupy multiple different belief states.

- *Compartmentalisation*: The contents of these different belief states (fragments) are independent.

- *Specialisation*: Different fragments control different aspects of behaviour / different practical domains.

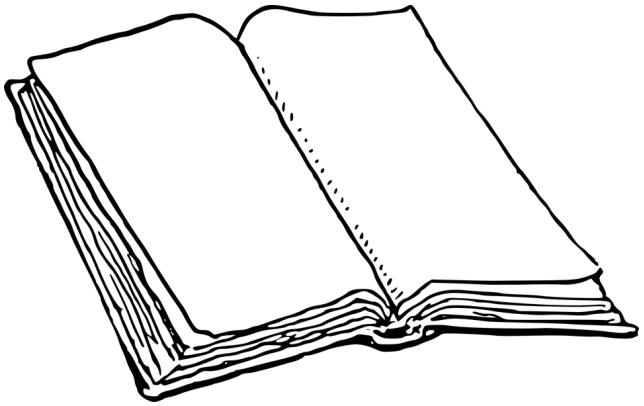
The Atlas



- A *fragmented belief state* $\mathbf{F}$ is a collection of classical information states $\{ \mathbf{D}_\alpha, \mathbf{D}_\beta, \ldots \}$

- An agent in state $\mathbf{F}$ *believes* *p* just in case *p* is true at all $\mathbf{D}_i$ worlds for some state $\mathbf{D}_i$ in $\mathbf{F}$.

The Atlas



- *Inconsistency*: $\mathrm{B}p \not\models \neg\mathrm{B}\neg p$
- *Adjunction failure*: $\mathrm{B}p, \mathrm{B}q \not\models \mathrm{B}(p \wedge q)$
- *Single-Premise Closure*: If $p \models q$, then $\mathrm{B}p \models \mathrm{B}q$
- *Non-Contradiction*: $\models \neg\mathrm{B}(p \wedge \neg p)$

The Atlas

- ✓ Allows for the possibility of *inconsistent beliefs*
- ✓ Allows for *failures of multi-premise closure*
- ✓ Makes independence between *recognition and recall* possible.

Three Problems

What about the virtues of the map?

? Can a fragmentation theorist explain how *belief predicts behaviour*?

? Can they account for *mind reading*?

? Can they account for the *dynamics of belief*?

Three Problems

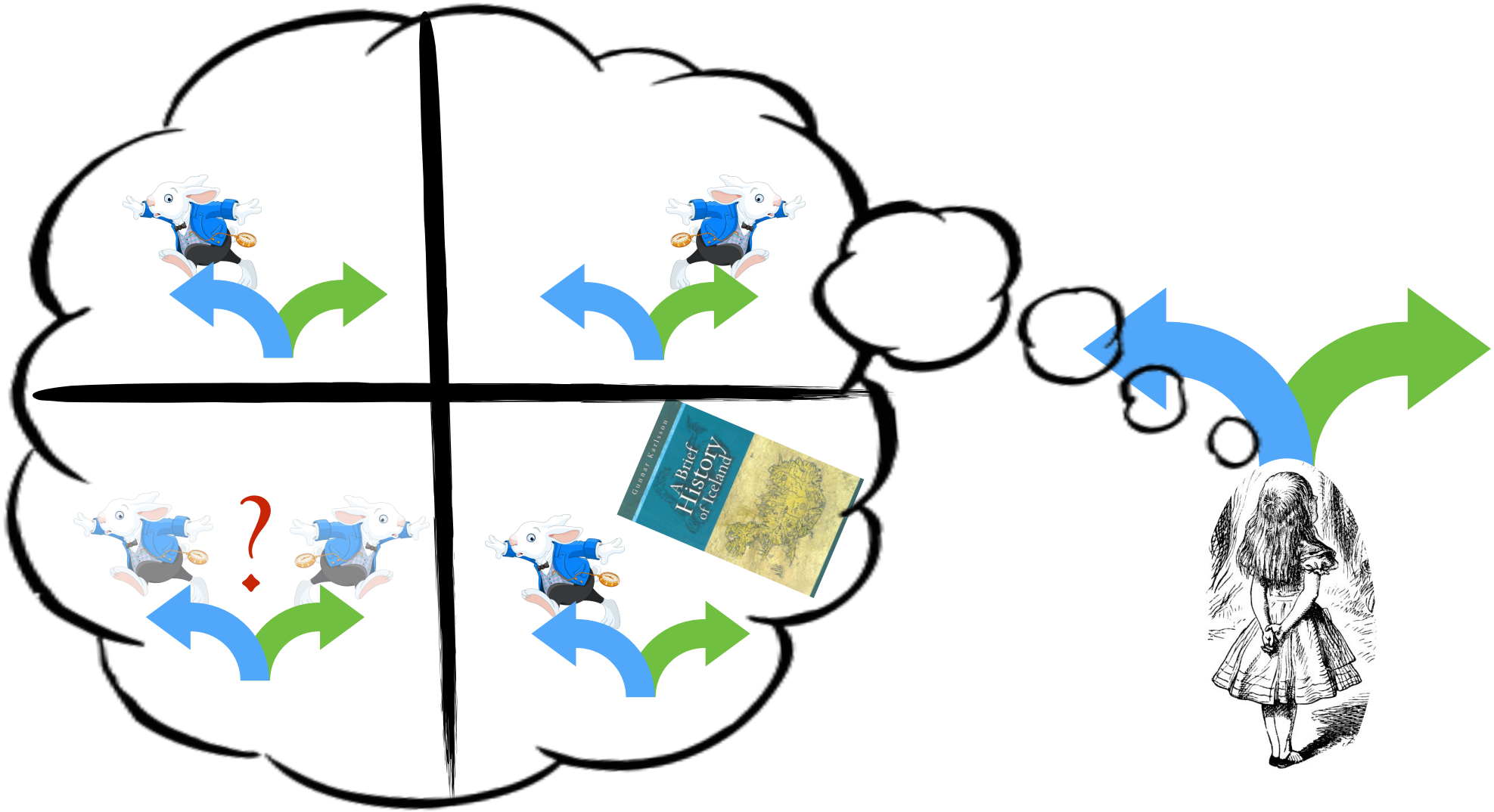
What about the virtues of the map?

? Problem of *Prediction*

? Problem of *Reading a Fragmented Mind*

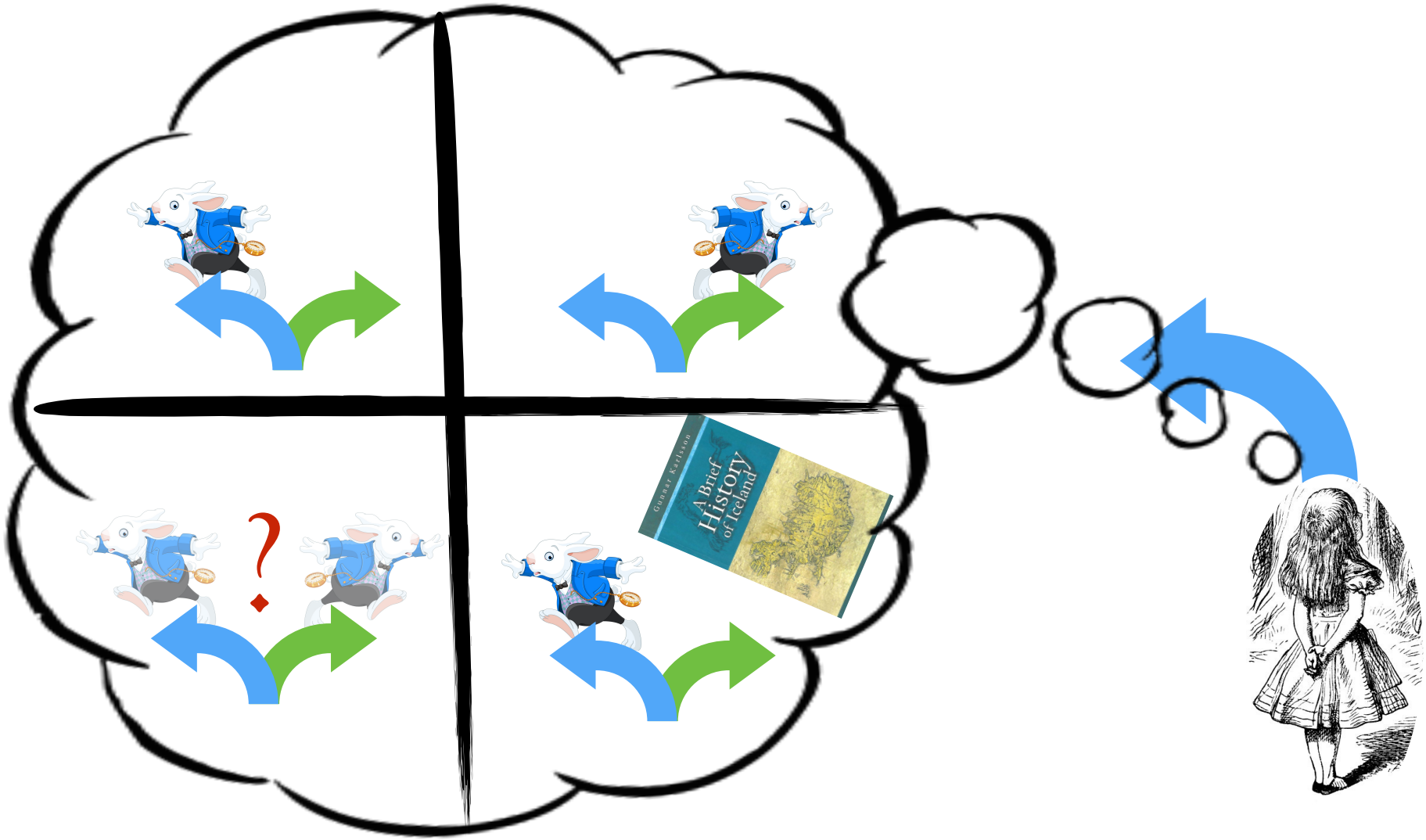
? Problem of *Learning*

The Problem of Prediction



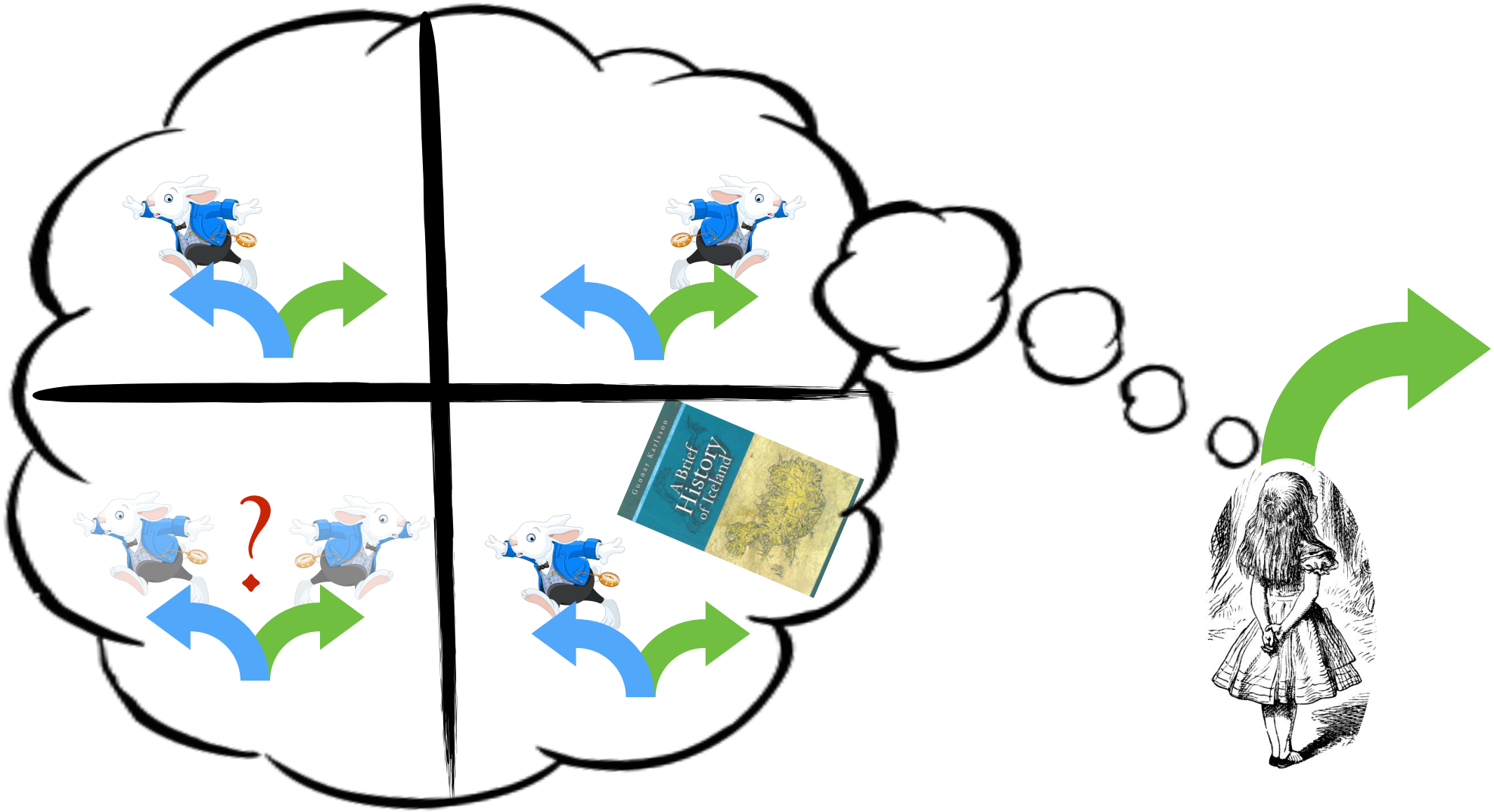
The Atlas by which we steer?

The Problem of Prediction



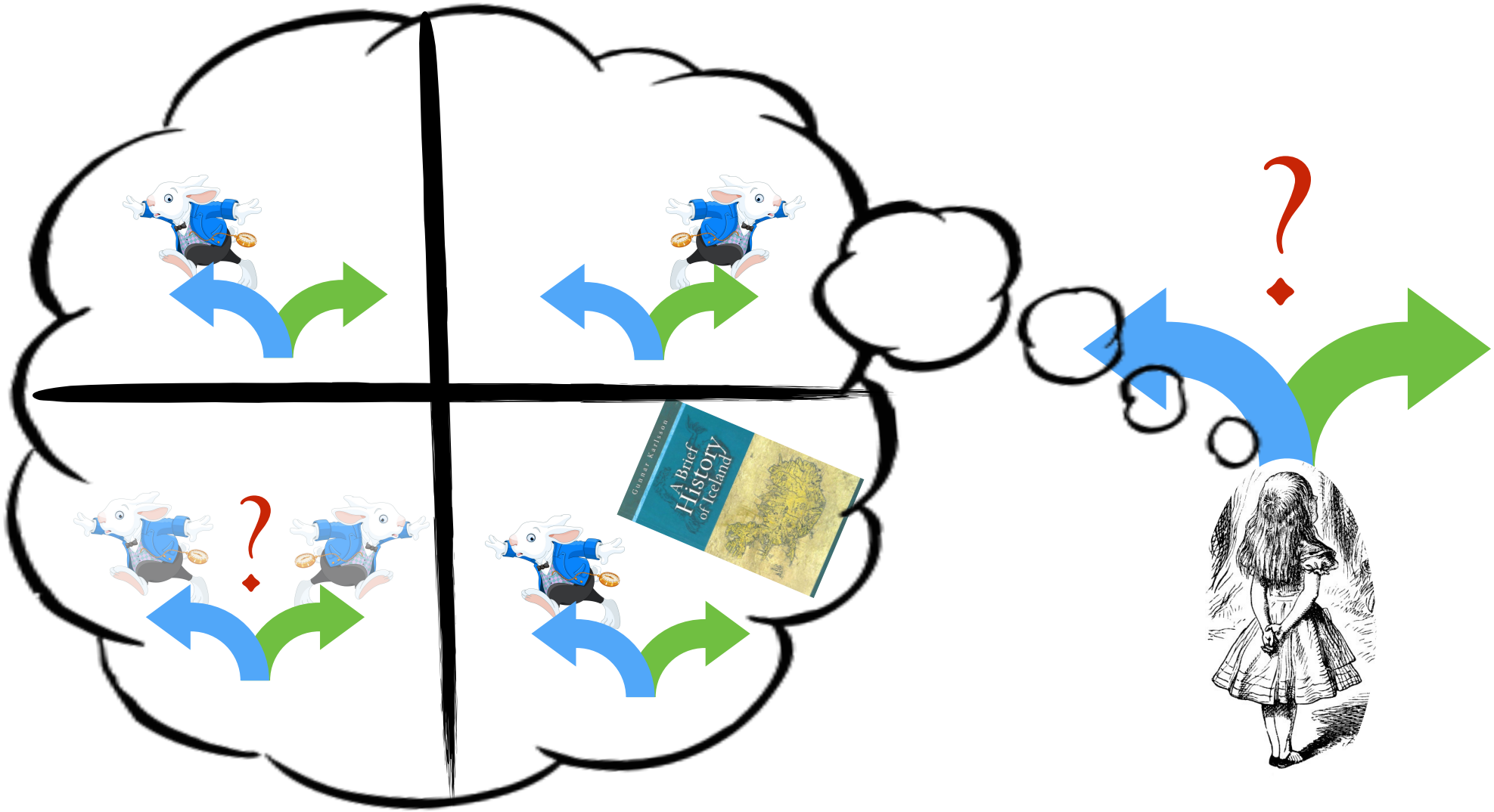
The Atlas by which we steer?

The Problem of Prediction



The Atlas by which we steer?

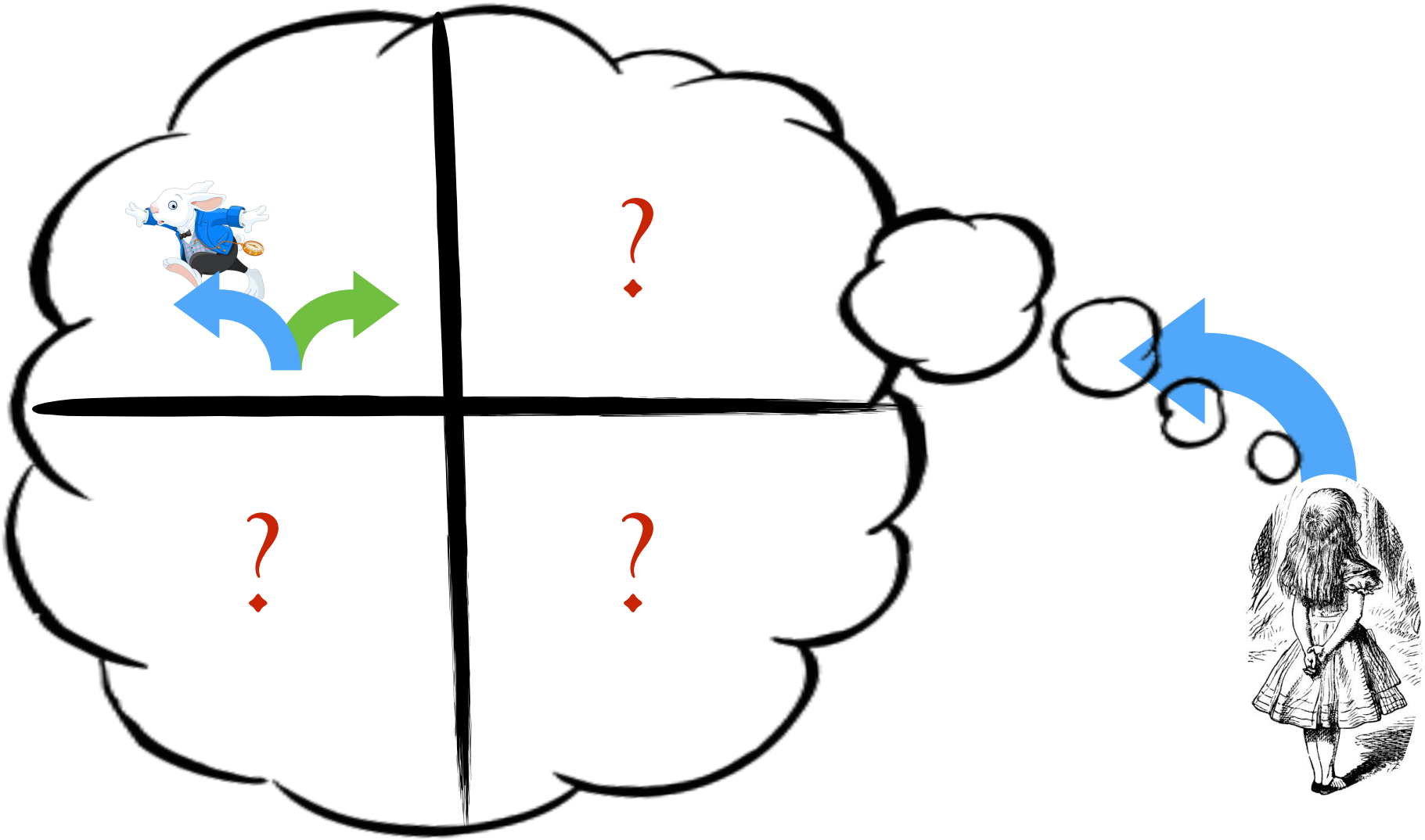
The Problem of Prediction



The Atlas by which we steer?

Reading a Fragmented Mind

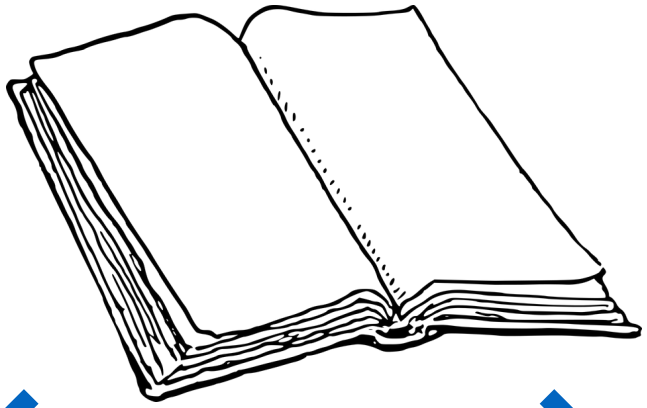
Reading a Fragmented Mind



The Problem of Learning

Basic questions about the dynamics of fragmented seem hard to answer in a principled way:

- If the agent learns some new piece of information, which fragment(s) is it added to?
- If the fragments are compartmentalised, how can evidence in one belief system be brought to bear on evidence in another fragment?
- How are multi-premise inferences made? Where do the conclusions go?



“One Map at a Time”

(Elga & Rayo 2021a)

“The Q&A Atlas”

(Yalcin 2011, 2018)

Elicitation Conditions

“One Map at a Time”

(Elga & Rayo 2021a)

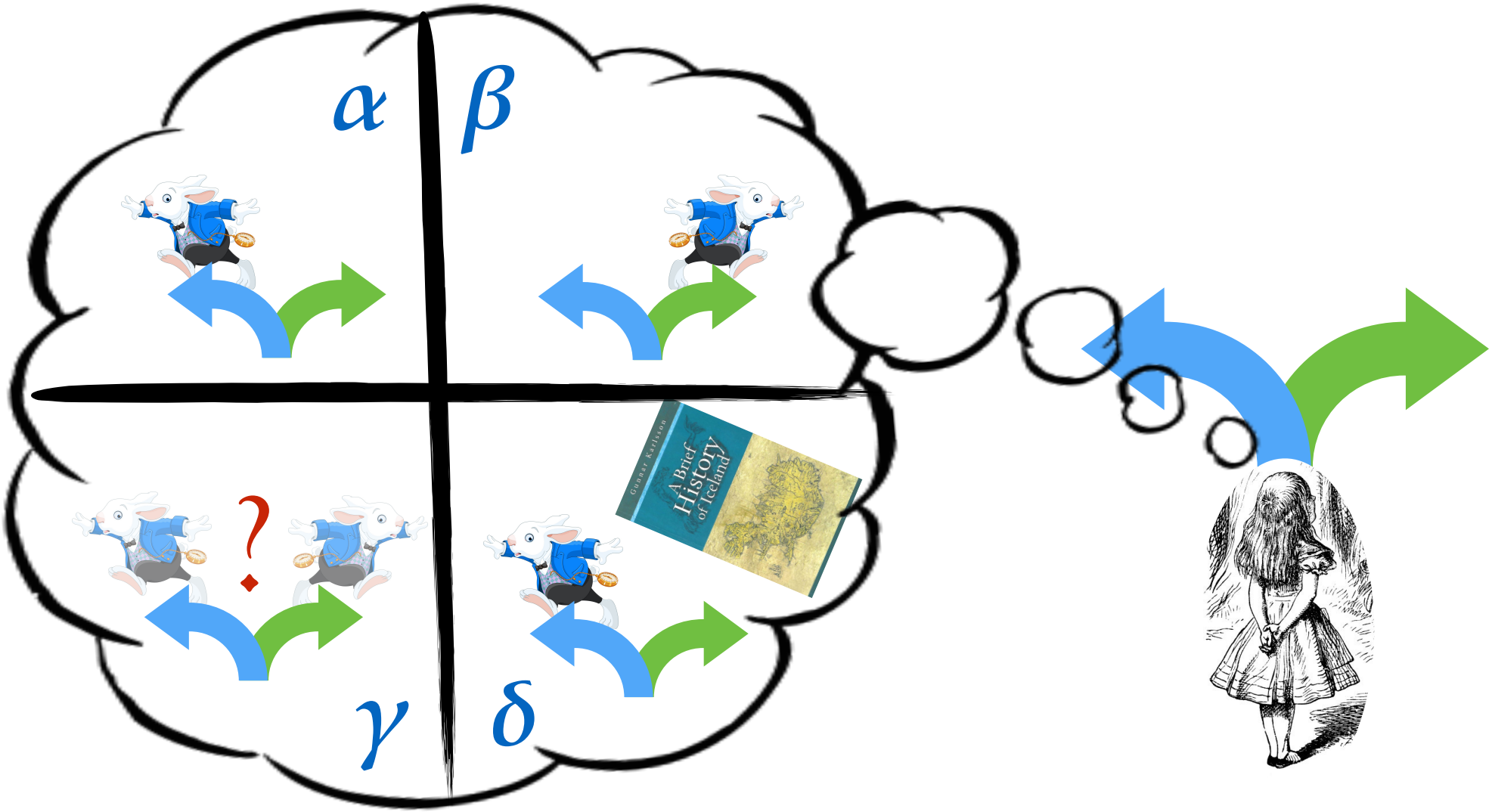
Elicitation Conditions

- An *access table* **A** is a collection of ordered pairs $\{ \langle \alpha, \mathbf{D}_\alpha \rangle, \langle \beta, \mathbf{D}_\beta \rangle, \ldots \}$. The second member of each pair is a belief state, and the first is the associated *elicitation condition*.

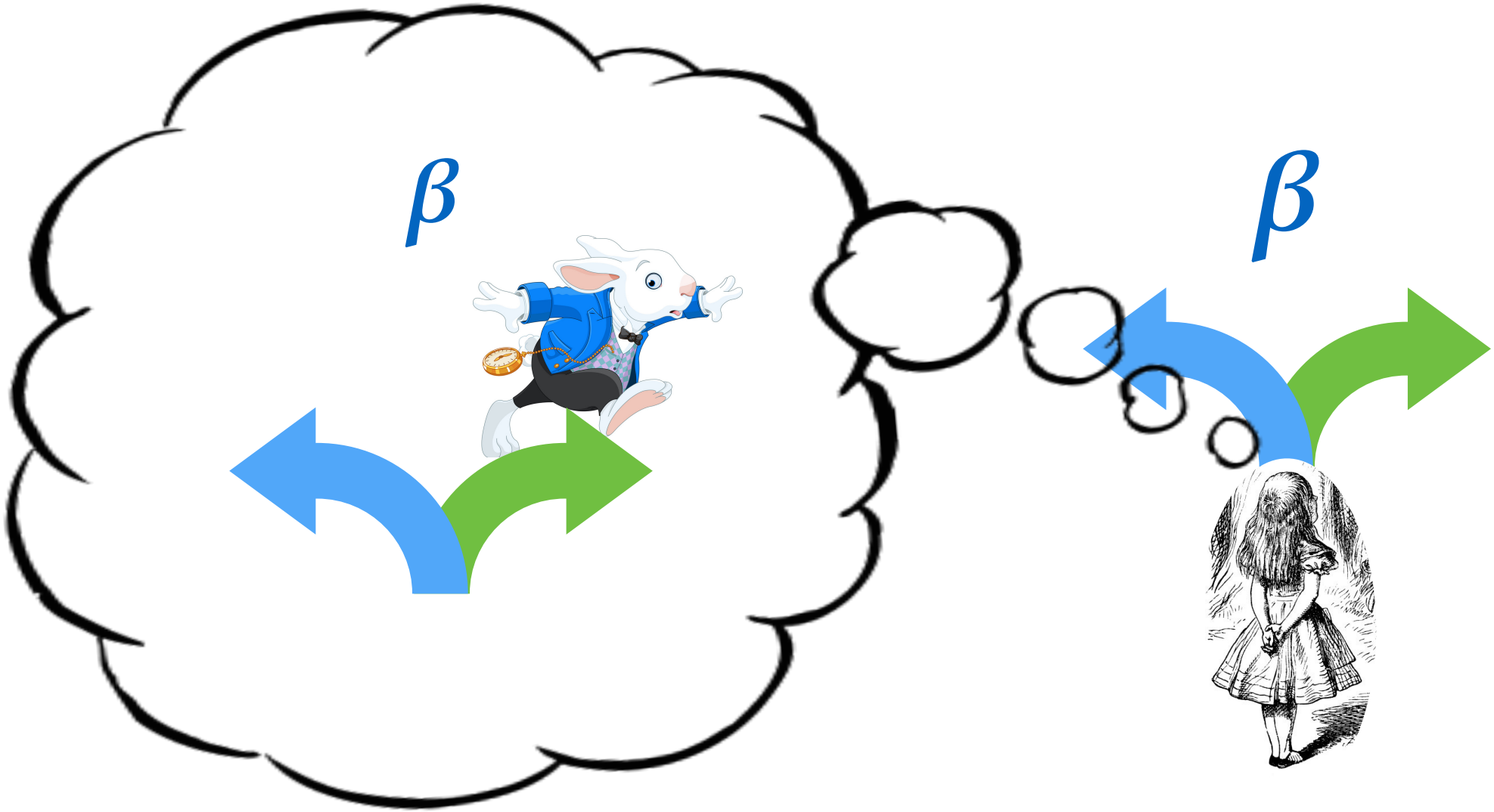
- The elicitation conditions partition the space of choices.

- *Fragmented Decision Rule*: In any situation where condition α obtains, an agent performs the action that would be most beneficial at $\mathbf{D}_\alpha$-worlds.

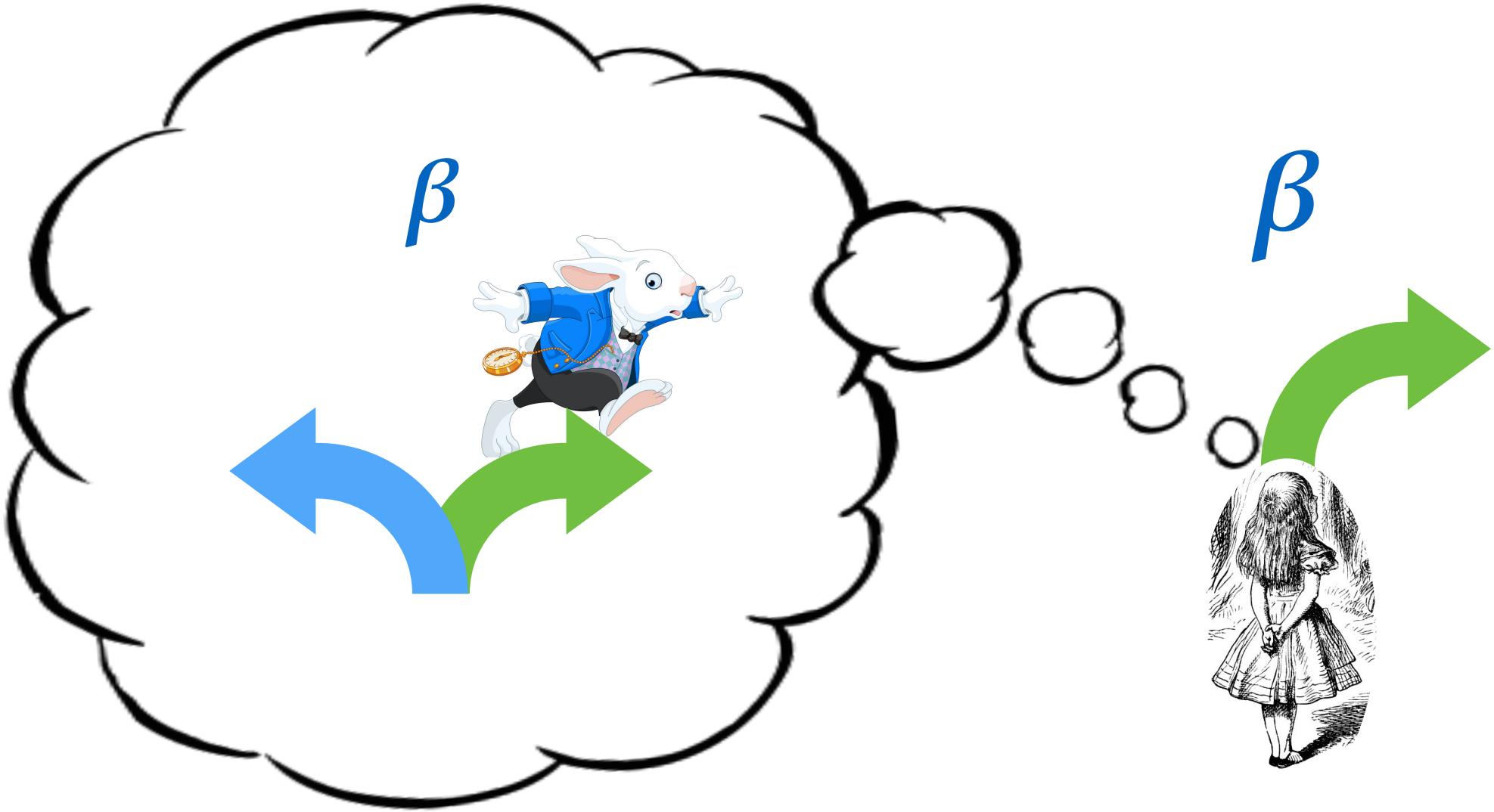
Prediction Problem (EC)



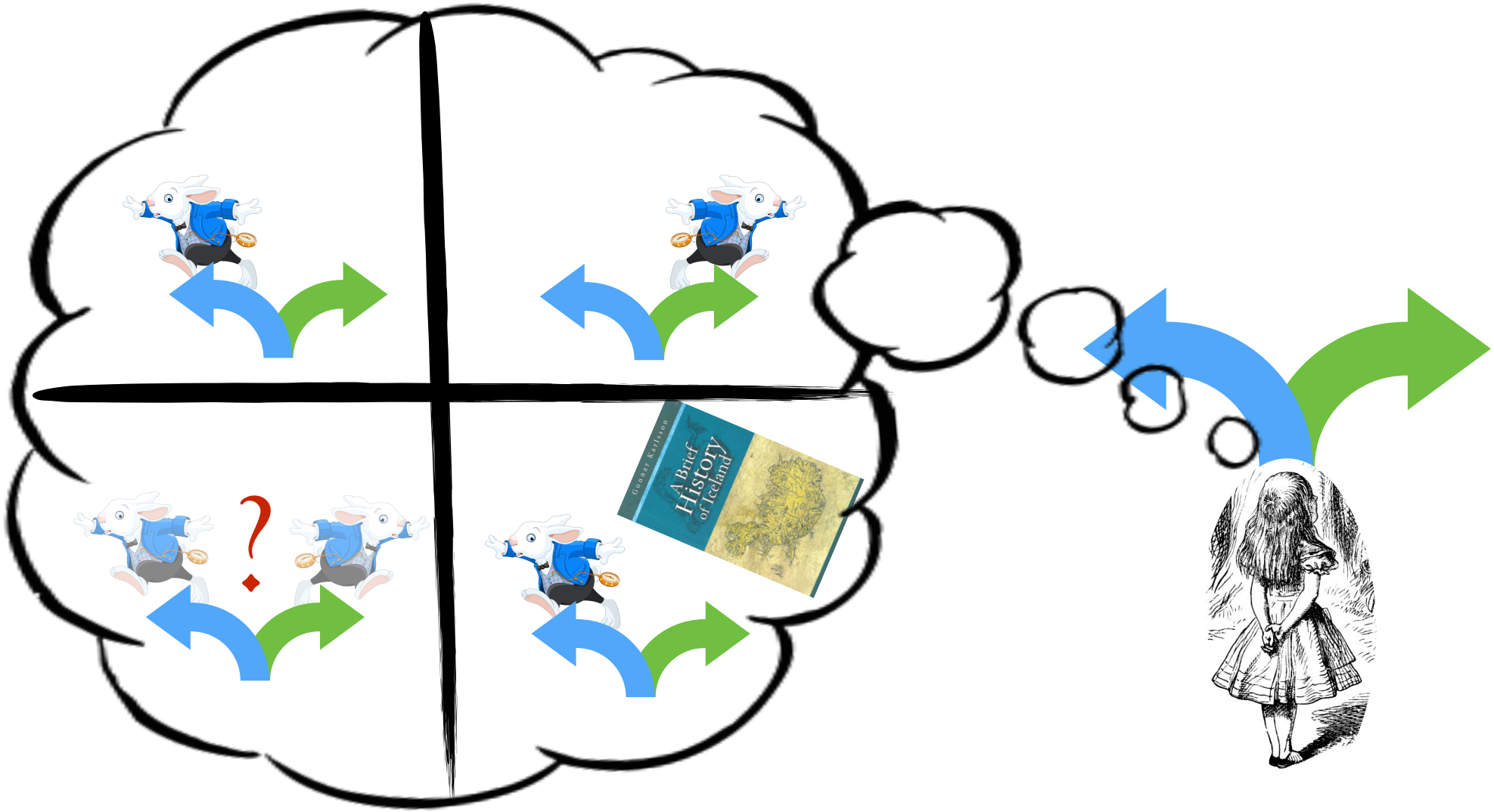
Prediction Problem (EC)



Prediction Problem (EC)



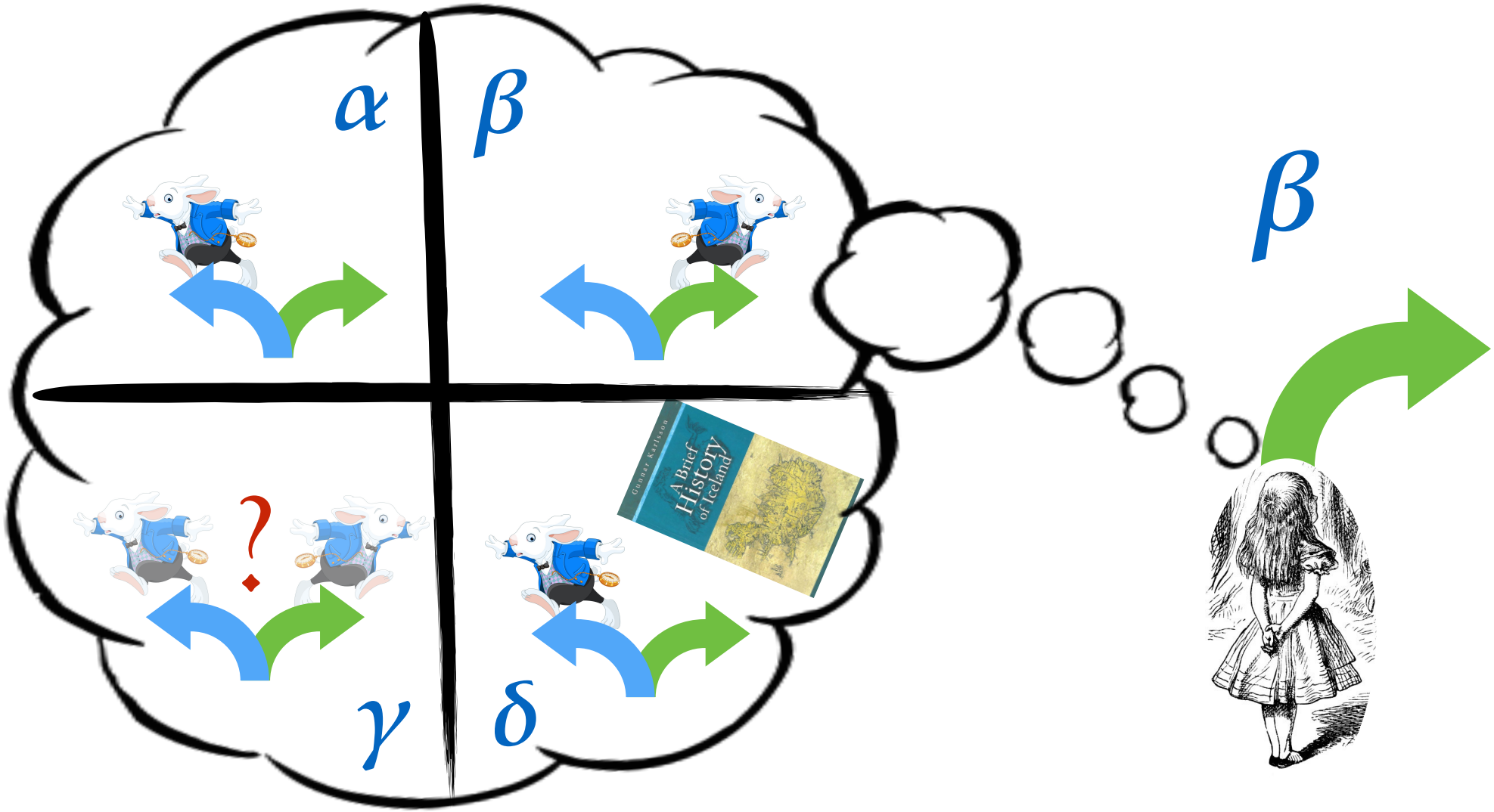
Prediction Problem (EC)



Prediction Problem (EC)

- Elicitation conditions are *not explanatory* (Norby 2014, Yalcin p.c.)
    - They just say *when* a fragment is active, not *why* it is active.
    - Compare an *action table* that "explains" an agents action by listing the action they would take in any decision situation.

Prediction Problem (EC)



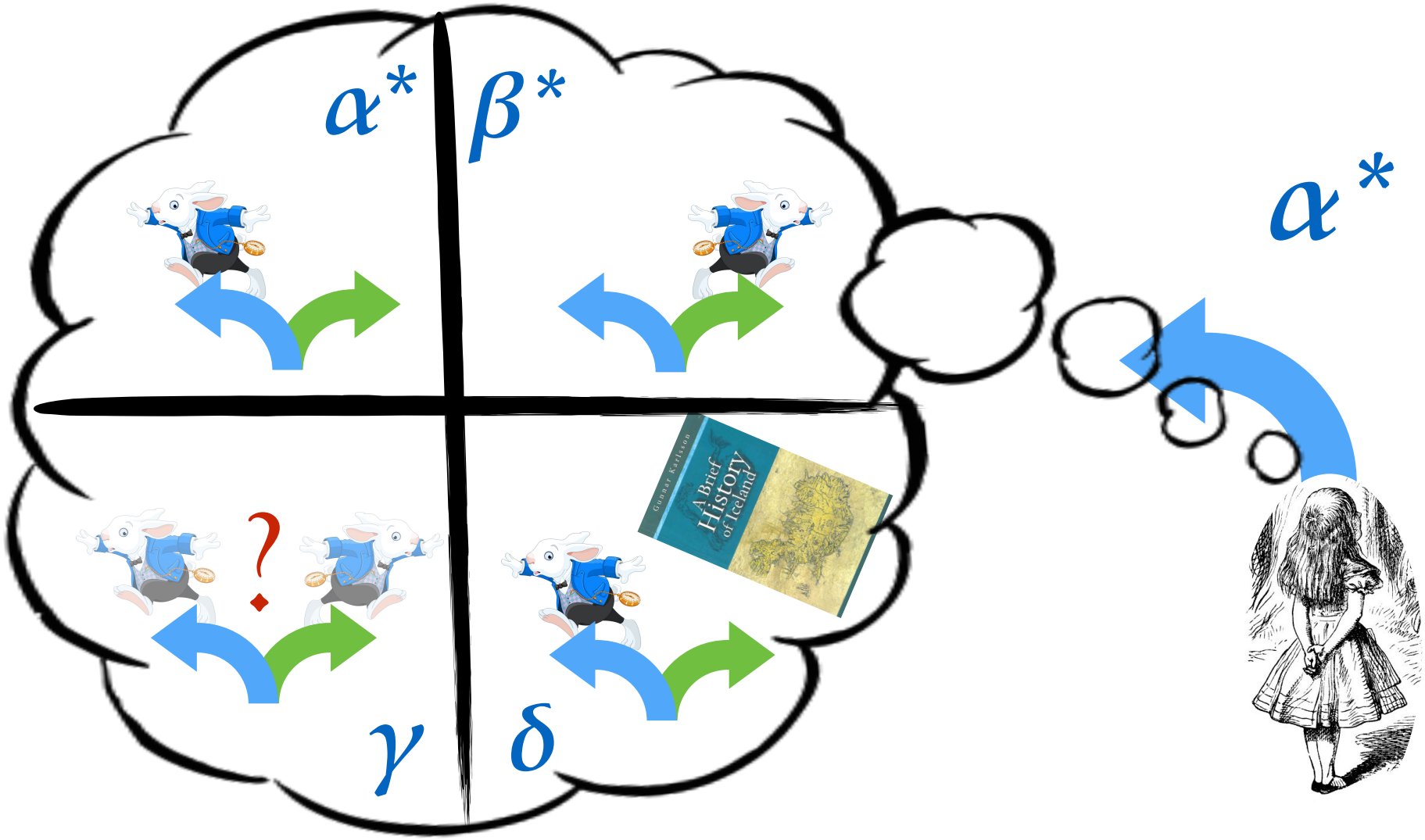
The Atlas by which we steer?

Prediction Problem (EC)

$$\alpha^* = \alpha \cup \{\Delta\}$$
$$\beta^* = \beta \setminus \{\Delta\}$$

Where Δ is the particular decision situation that Alice is in.

Prediction Problem (EC)

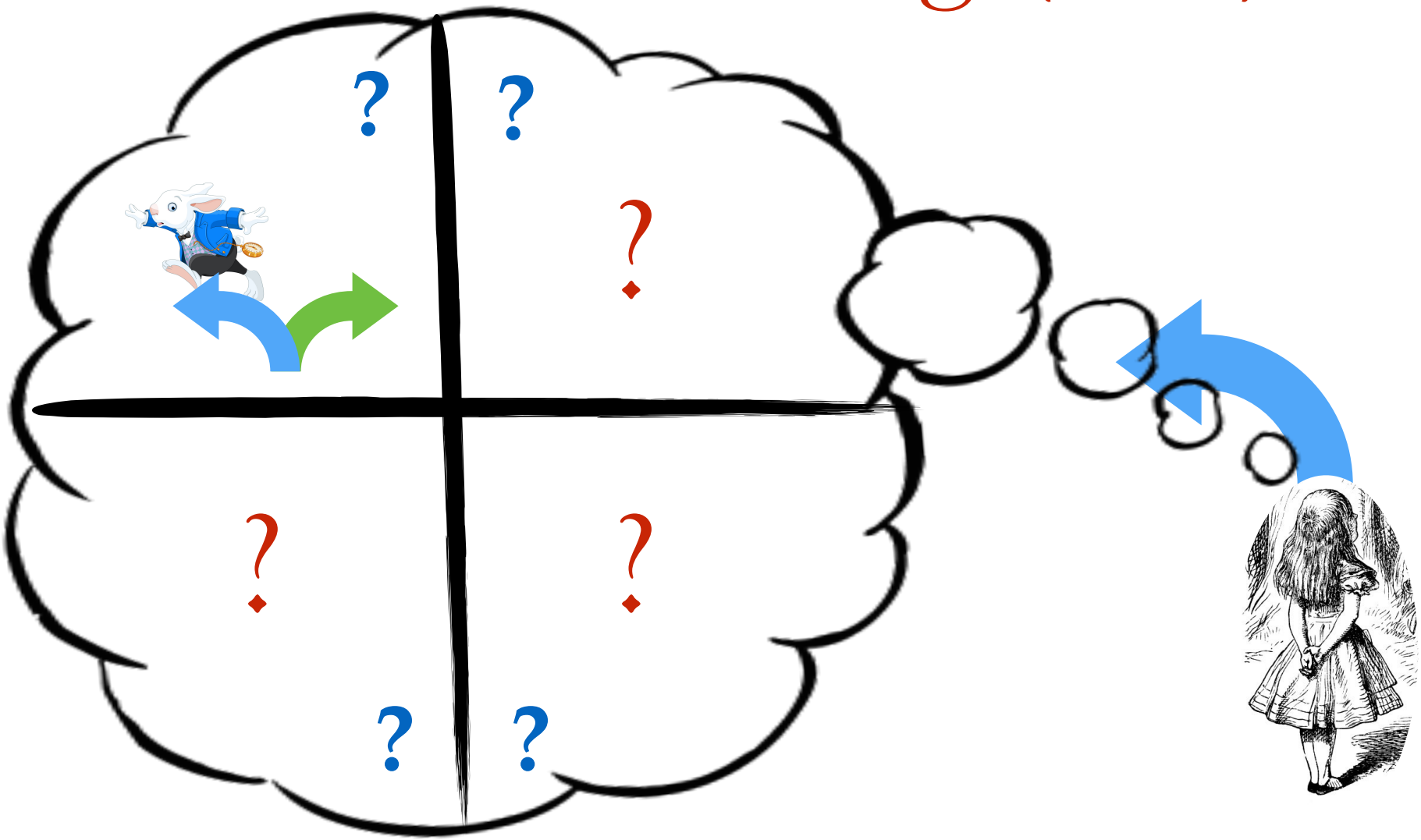


The Atlas by which we steer?

EC Scorecard

- ✗ Unsatisfactory explanation of how *belief predicts behaviour*

Mind Reading (EC)



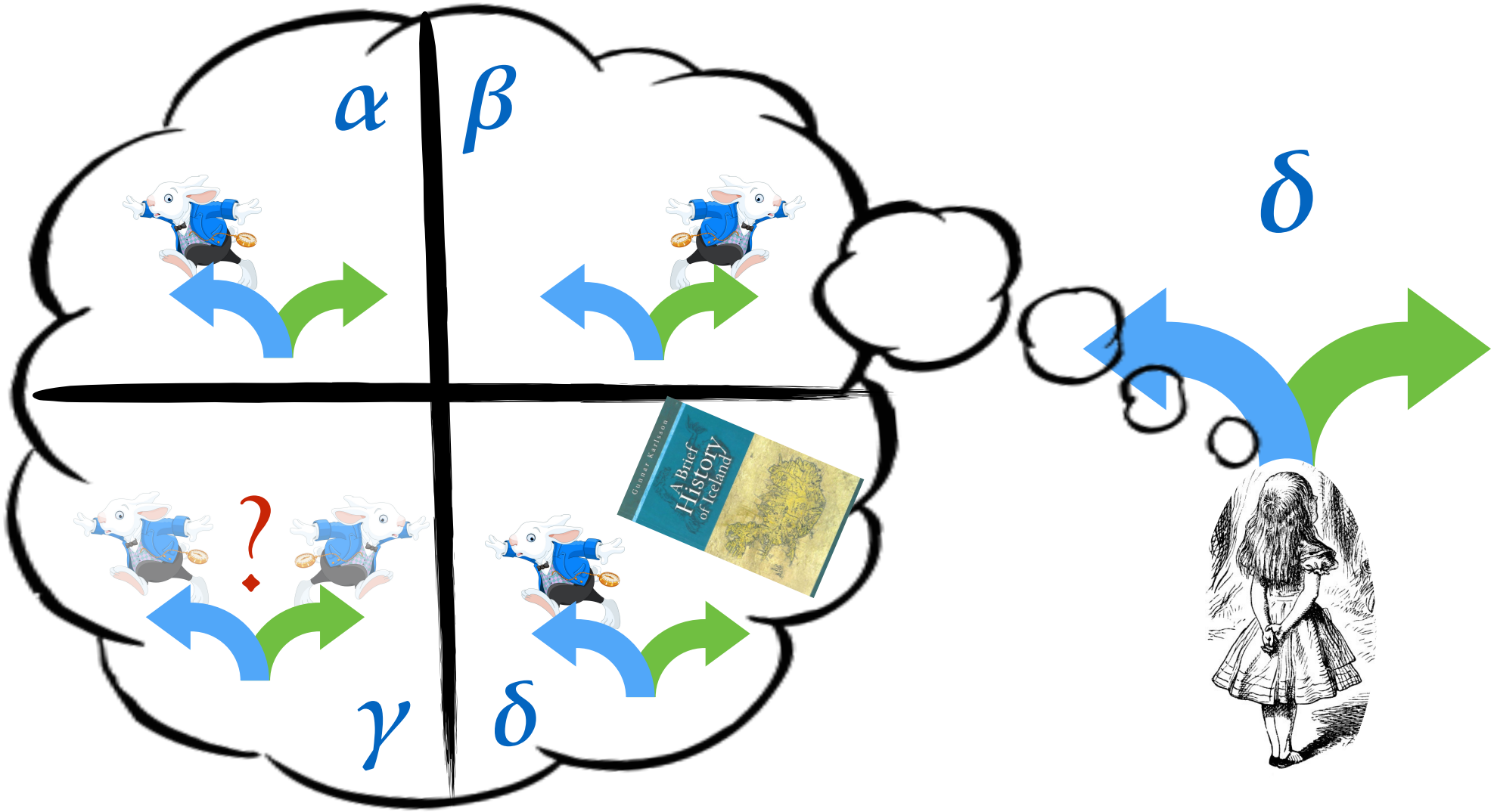
Mind Reading (EC)

- No account of negative belief attributions.
- Mysterious how elicitation conditions can come to be known.

EC Scorecard

- ✗ Unsatisfactory explanation of how *belief predicts behaviour*
- ✗ Partial account of *mind reading*
- ✗ Elicitation conditions don't answer any questions about the *dynamics of belief.*

Problem of Inert Beliefs



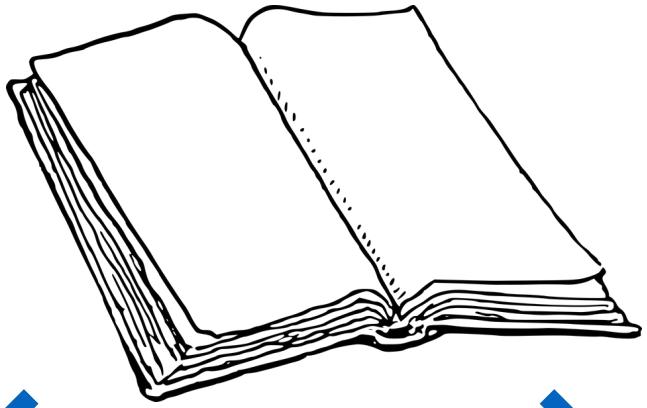
Problem of Inert Beliefs

Problem of Inert Beliefs

- The fragmentation picture allows for beliefs that lack any functional role: they are not “linked up” to either the world or to the agent’s behaviour.
- The existence of such beliefs can’t be squared with a functionalist picture of mind.
- This makes a fragmented representation theorem impossible in principle.
    - The issue is that an agent’s preferences between options (their behavioural dispositions) underdetermine their beliefs.

EC Scorecard

- ✗ Unsatisfactory explanation of how *belief predicts behaviour*
- ✗ Partial account of *mind reading*
- ✗ Elicitation conditions don't answer any questions about the *dynamics of belief.*
- ✗ No *representation theorem* is possible.



“One Map at a Time”

(Elga & Rayo 2021a)

“The Q&A Atlas”

(Yalcin 2011, 2018)

Question-Sensitivity

“The Q&A Atlas”

(Yalcin 2011, 2018)

Question-Sensitivity

- An *doxastic state* $\mathbb{Q}$ is a collection of ordered pairs $\{ \langle \mathbf{Q}_\alpha, \mathbf{A}_\alpha \rangle, \langle \mathbf{Q}_\beta, \mathbf{A}_\beta \rangle, \ldots \}$. The first member of each pair is a *question*, and the second member is a *(partial) answer* to that question.
    - No question appears twice — $\mathbb{Q}$ is a *function* from questions to answers.
    - Formally, each question $\mathbf{Q}_i$ is a partition of logical space, and each answer $\mathbf{A}_i$ a union of $\mathbf{Q}_i$-cells.
- An agent in state $\mathbb{Q}$ *believes* p just in case for some $\langle \mathbf{Q}_i, \mathbf{A}_i \rangle \in \mathbb{Q}$, p is addressed at $\mathbf{Q}_i$, and $\mathbf{A}_i$ entails p.

Question-Sensitivity

An agent in state $\mathbb{D}$ *believes* p just in case for some $\langle \mathbf{Q}_i, \mathbf{A}_i \rangle \in \mathbb{D}$, p is addressed at $\mathbf{Q}_i$, and $\mathbf{A}_i$ entails p.

- Like beliefs, the prejacents in belief reports are addressed at specific questions. (Yalcin 2011)
- *No Single-Premise Closure*: If p and q address different questions, $\mathrm{B}p \not\models \mathrm{B}q$
- *No Conjunction Elimination*: $\mathrm{B}(p \wedge q) \not\models \mathrm{B}q$

Four Problems

- ? Problem of *Prediction*
- ? Problem of *Reading a Fragmented Mind*
- ? Problem of *Learning*
- ? Problem of *Inert Beliefs*

Four Problems

- ✗ Problem of *Prediction*
- ✗ Problem of *Reading a Fragmented Mind*
- ? Problem of *Learning*
- ? Problem of *Inert Beliefs*

Problem of Learning (QS)

If the agent learns some new piece of information, which fragment(s) is it added to?

- Belief is the product of ***inquiry*** into particular questions (Friedman 2017, 2020; Drucker 2020; Peirce 1877).

- Our beliefs are the answers to the questions that guided the inquiry giving rise to them.

Problem of Learning (QS)

Who has my flamingo?

Is it a courtier?

Is it the Cheshire cat?

Is it Alice?

Problem of Learning (QS)

- The question **Q′** is *part of* **Q** just in case every answer to **Q′** is a partial answer to **Q**.
    - Informally, **Q′** is part of **Q** just in case you need to find out the answer to **Q′** in order to find out the complete answer to **Q**.
- When we inquire into a big, complex question with many parts, we often do so by inquiring into the parts first.

Problem of Learning (QS)

But the atlas picture makes it mysterious why inquiry into a big question **Q** can be conducted by means of inquiry into a part **Q′** of **Q**.

- On the face of it, the atlas picture suggests that only inquiry into **Q** itself can lead to new beliefs / discoveries about **Q**.

- Given compartmentalisation, how does an agent's view on **Q′** affect their view on **Q**?

QS Scorecard

✗ No decision rule, so no solution to the *prediction problem*

✗ Likewise, no solution to the *mind reading problem*

✓ We have an account of *learning* through inquiry, though complex inquiries still raise hard questions.

Problem of Inert Beliefs (QS)

- *Belief-World Connection*. Assuming any question we have views on can be the object of inquiry, the inquiry story links all beliefs to the world.

- *Belief-Action Connection*. Assuming any question we have views on can be practically relevant, any answer we have can make a practical difference.

    - (This assumes that our views on a given question influence behaviour in decision situations where those questions are practically relevant.)

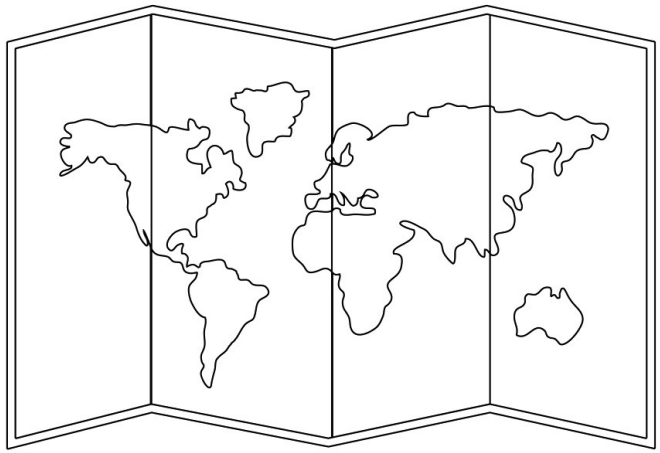
QS Scorecard

✗ Absent a decision rule, there's no solution to the *prediction problem*

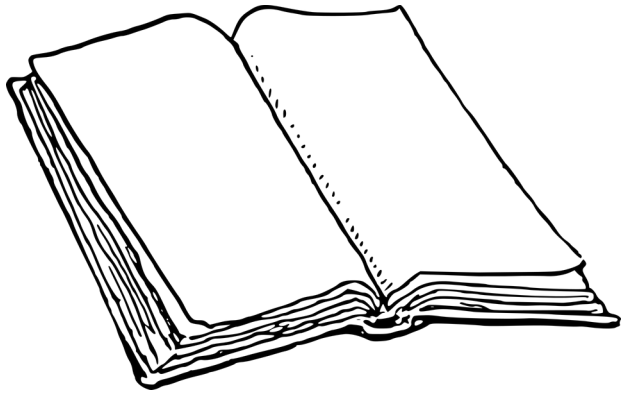
✗ Likewise, no solution to the *mind reading problem*

✓ We have an account of *learning* through inquiry, though complex inquiries still raise hard questions.

✓ We have a solution to the *problem of inert beliefs*, removing one obstacle to a functionalist account of beliefs (and to a representation theorem).



“The Map”



“The Atlas”

“The Web”

Choices and Questions

- *Choices confront us with questions*.

- Formally, the question a decision situation confronts you with is just the *world state partition* of the decision situation. (Hoek 2019)

	Rabbit turned left	*Rabbit turned right*
Turn left	Find the rabbit	Lose the rabbit
Turn right	Lose the rabbit	Find the rabbit

Choices and Questions

- In our discussion of the Problem of Inert beliefs, we said that question-sensitive beliefs should be expected to guide our choices in situations where the question is practically relevant.

- That suggests a *Question-Sensitive Decision Rule*:

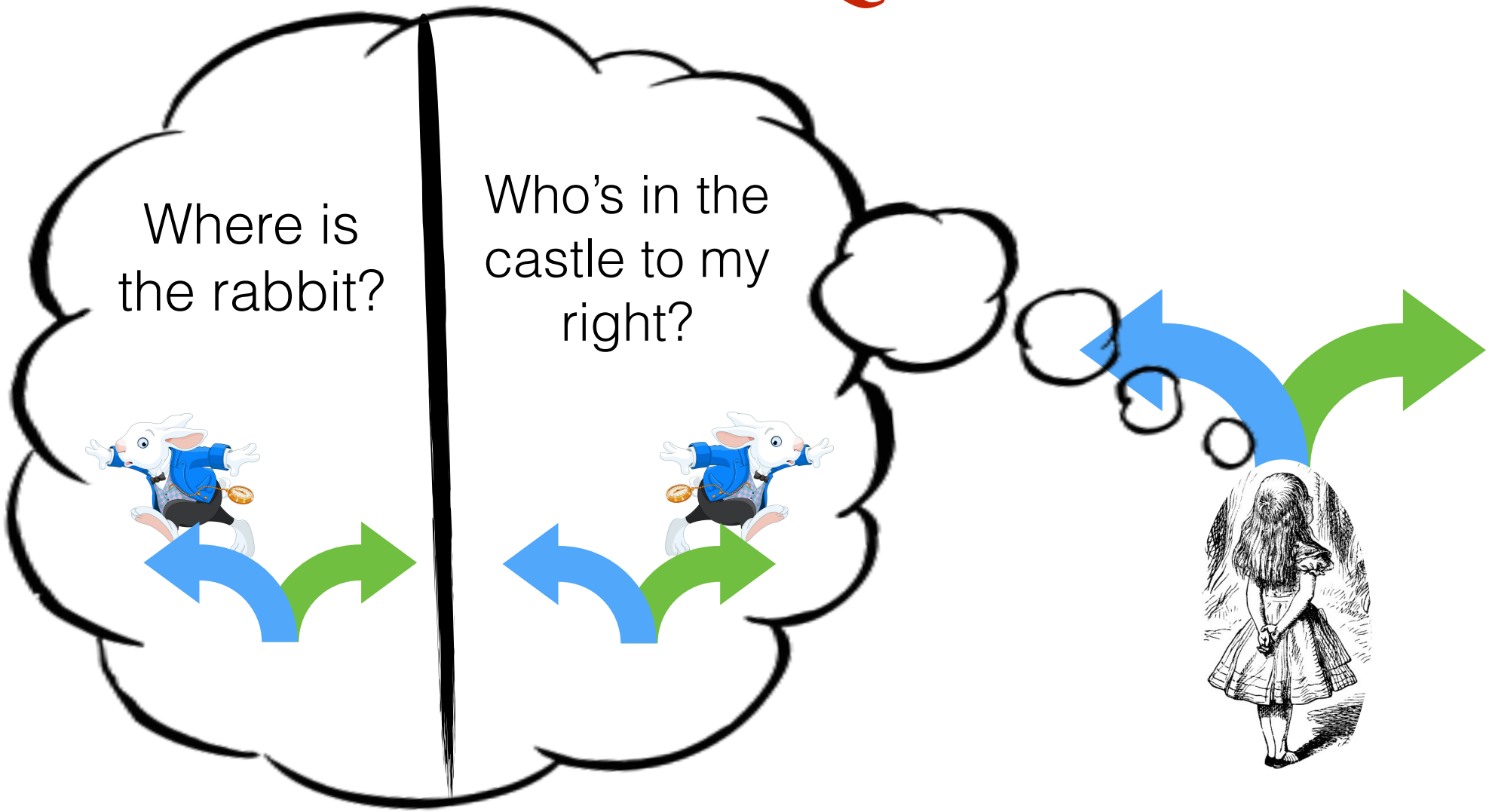
    - When an agent is confronted with the question $\mathbf{Q}_\alpha$, the agent performs the action that would be most beneficial at $\mathbf{A}_\alpha$-worlds.

Choices and Questions

What is Alice's number?

To whom does the number 529 6300 belong?

Choices and Questions



Harmony Between the Parts

- Alice believes that Charlie's *house number* is forty-three.
- She also believes that Charlie's *address* is fifty-two Turl Street.
- Suppose Alice is asked to write down Charlie's address. What will she write?

Harmony Between the Parts

- As with inquiry, the difficulties arise when there is a mismatch between our view on a big question, and our view on its parts.

- To avoid these issues, why not impose the constraint that *a view on a question and the views on its parts should always match*?

- *Harmony*: If an agent has a view on **Q**, and **Q′** is part of **Q**, then their view on **Q′** is the union of all **Q′**-cells consistent with their view on **Q**.

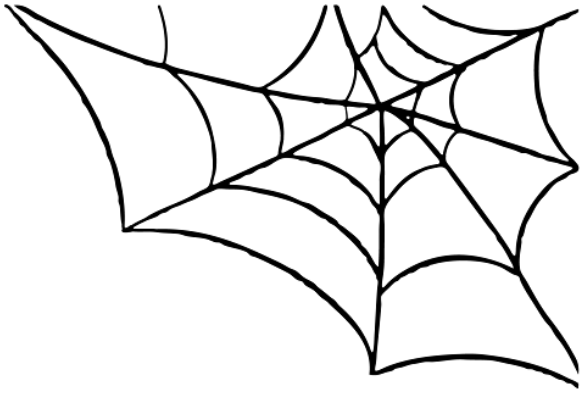
Harmony Between the Parts

- *Propositional Mereology* Propositions have *parts*. (Gemes 1994, Yablo 2014, Fine 2017)
- *Doxastic Mereology*: Agent's beliefs have *parts* which are themselves also views the agent holds. (Yablo 2014, Hawke 2016)
- *Inquisitive Doxastic Mereology*: The parts of an agent's view on some question **Q** are their views on the parts of **Q**.

Harmony Between the Parts

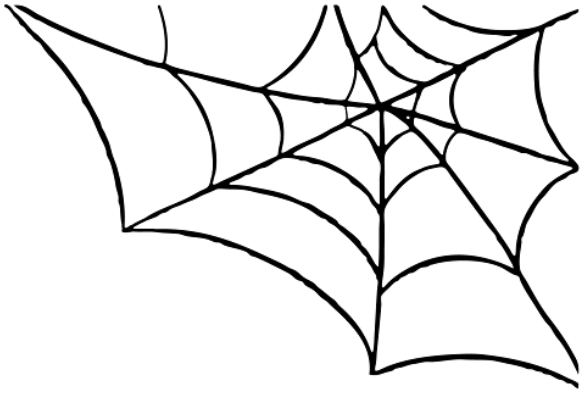
- Views may also concern *overlapping questions*.
- By *Harmony*, views on overlapping questions must match on the overlapping part.
- But then indirect mereological connections emerge even between disjoint views: they may be connected by a daisy-chain of views, with each link in the chain overlapping the adjacent views.

The Web



- *Doxastic Mereology*: Agents' views have *parts* which are themselves also views the agent holds.
- *Holism*: Even disjoint views may be indirectly mereologically related, so that information may in principle flow throughout the web.
- *Generality*: Beliefs guide all choices the same way, regardless of their practical domain.

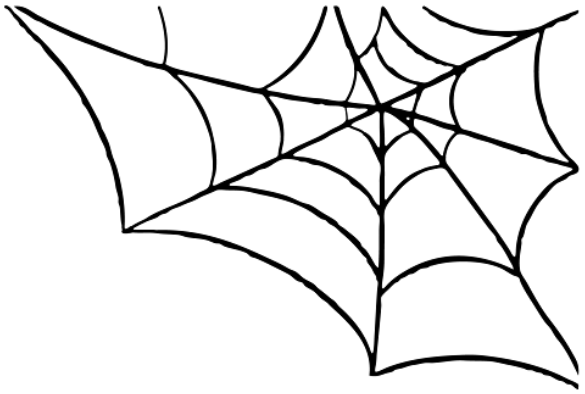
The Web



- A *belief web* $\mathbb{W}$ is a set of question-answer pairs $\{ \langle \mathbf{Q}_\alpha, \mathbf{A}_\alpha \rangle, \langle \mathbf{Q}_\beta, \mathbf{A}_\beta \rangle, \ldots \}$ that satisfies the *Harmony* constraint.

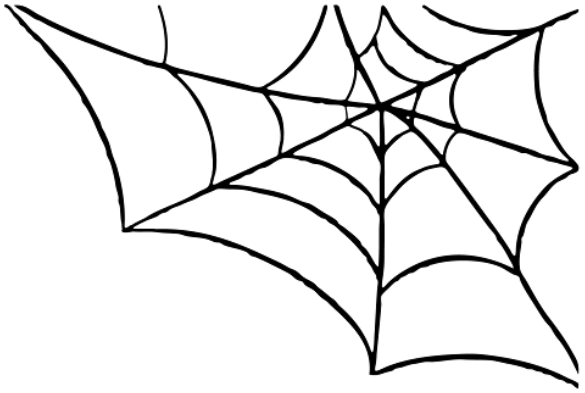
- An agent in state $\mathbb{D}$ *believes* p just in case for some $\langle \mathbf{Q}_i, \mathbf{A}_i \rangle \in \mathbb{D}$, p is addressed at $\mathbf{Q}_i$, and $\mathbf{A}_i$ entails p.

The Web



- *No Single-Premise Closure*: $\mathrm{B}p \not\models \mathrm{B}(p \vee q)$
- *Closure under Parthood*: $\mathrm{B}(p \wedge q) \models \mathrm{B}p$
- *Coherence*: $\mathrm{B}p \models \neg\mathrm{B}\neg p$
- *Partial Adjunctive Closure*: $\mathrm{B}(p \wedge (q \vee \neg q)), \mathrm{B}q \models \mathrm{B}(p \wedge q)$

The Web



- *Inquisitive belief updates*: $\mathbb{W}$ + ⟨**Q**, **A**⟩ is defined as the smallest web containing all the beliefs in $\mathbb{W}$ and also ⟨**Q**, **A**⟩

- *Tautological updates*: Much deductive reasoning, including multi-premise inference, can be modelled in terms of updates by tautological propositions. (Hoek 2020)

The Web: Scorecard

- ✓ Explains how *question-sensitive belief predicts behaviour*
- ✓ Accounts for *question-sensitive mind reading*
- ✓ Yields theories of *learning* and *deductive inference*
- ✓ Fits into a *functionalist view of the mind* (and there is a representation theorem)

The Web: Scorecard

✓ Allows behaviour prediction for agents with *inconsistent beliefs* or *closure failures*

✓ Explains the difference between *recognition and recall* in terms of a hyperintensional difference in belief content.

✗ The account of updates works only for full belief. I have no account of *inquisitive conditionalisation*

✗ There is also no account of *belief revision*.

The Web

"The inquiries we confront are obviously myriad and heterogenous:

Are we heading towards a recession? Can you sleep-train a four-month-old? Why aren't my tomatoes growing faster? Where can I get a taco at this hour?…"

The Web

"*How much sunscreen to use? Is Pluto a planet? Is fragmentation rational? What's the deal with airplane food?*

The idea that there is rational pressure on me … to bring together all my thinking in these directions into one super rich logical space, seems bizarre on its face. If anything, it seems pathological to seek this kind of unity." (Yalcin 2021)

Thank You!

dhoek@vt.edu

References

Borgoni, Cristina, Dirk Kindermann and Andrea Onofri, *The Fragmented Mind*, Oxford: Oxford University Press 2021.

Drucker, Daniel, 2020, "The Attitudes We Can Have." *The Philosophical Review* 129(4): 591-642.

Egan, Andy, 2008, "Seeing and believing: perception, belief formation and the divided mind." *Philosophical Studies* 140(1): 47–63

Elga, Adam, and Agustín Rayo, 2021a. "Fragmentation and information access." Borgoni et al.

Elga, Adam, and Agustín Rayo, 2021b. "Fragmentation and logical omniscience." *Noûs*, online first.

Fagin, Ronald, and Joseph Y. Halpern, 1988, "Belief, Awareness, and Limited Reasoning." *Artificial Intelligence* 34: 39–76.

Fine, Kit, 2017, "A Theory of Truthmaker Content I, II" Journal of Philosophical Logic 46(6): 625–702. Friedman, Jane,

Friedman, Jane, 2013, "Question-Directed Attitudes." *Philosophical Perspectives* 27(1): 145-74.

Friedman, Jane, 2017, "Inquiry and Belief." *Noûs* 53(2): 296-315.

Gemes, Ken, 1994, "A new theory of content I: Basic content." *Journal of Philosophical Logic*, 23(6):595–620.

Gemes, Ken, 1997, A new theory of content II: Model theory and some alternatives. *Journal of Philosophical Logic*, 26(4):449–476, 1997.

References

Greco, Daniel, 2015, “Iteration and Fragmentation.” *Philosophy and Phenomenological Research* 91(3): 656-673

Hawke, Peter, 2016, “Questions, topics, and restricted closure.” *Philosophical Studies* 173(10): 2759-84.

Hintikka, Jaakko, 1992, Knowledge and Belief. Ithaca: Cornell University Press.

Hoek, Daniel, 2019, *The Web of Questions: Inquisitive Decision Theory and the Bounds of Rationality*. PhD Dissertation, New York University.

Hoek, Daniel, 2020, “Minimal Rationality and the Web of Questions.” To appear in Kindermann et al. (eds.), *The Fragmented Mind*, Oxford University Press.

Hoek, Daniel and Richard Bradley, ms., “Million Dollar Questions: Why Deliberation is More than Information Pooling.” Draft dated August 2021.

Koralus, Philipp, 2014, “The Erotetic Theory of Attention: Questions, Focus and Distraction.” *Mind & Language* 29.1: 26-50.

Koralus, Philipp, and Salvador Mascarenhas, 2013, “The Erotetic Theory of Reasoning” *Philosophical Perspectives* 27(1): 312-365.

Levesque, Hector J., 1984, “A logic of implicit and explicit belief.” *AAAI-84 Proceedings*, 198-202.

Lewis, David, 1982, “Logic for Equivocators.” *Noûs* , Sep., 1982, Vol. 16, No. 3 (Sep., 1982), pp. 431-441

Peirce, C.S.,1877, “The Fixation of Belief.” *Popular Science Monthly* 12: 1-15.

References

Ramsey, Frank, 1926, “Truth and Probability.” In Ramsey 1931, 138–154.

1929, “General Propositions and Causality.” In Ramsey 1931, 237–255.

Rescher, Nicholas and Robert Brandom, 1980, *The Logic of Inconsistency*. Oxford: Blackwell.

Stalnaker, Robert, 1984, *Inquiry*. Cambridge, MA: MIT Press.

Stalnaker, Robert, 1991, “The Problem of Logical Omniscience I.” Repr. in *Context and Content*, 241-54, New York: Oxford University Press.

Stalnaker, Robert, 1999, “The Problem of Logical Omniscience II.” Repr. in *Context and Content*, 255-73, New York: Oxford University Press.

Yablo, Stephen, 2014, *Aboutness*. Princeton, NJ: Princeton University Press.

Yablo, Stephen, 2017, “Open knowledge and changing the subject.” *Philosophical Studies* 174(4): 1047-71.

Yalcin, Seth, 2011, “Nonfactualism About Epistemic Modality.” In *Epistemic Modality*, eds. Andy Egan and Brain Weatherson, 295-332. New York: Oxford University Press.

2018, “Belief as Question-Sensitive.” *Philosophy and Phenomenological Research*, 97(1): 23-47.

2021, “Fragmented But Rational.” In Borgoni et al.