The Fourth-Dimensional Twist
or
A Modest Proposal in Aid of the American Driver in England *

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An American driving in England is confronted with a wild and dangerous world. The cars have the driver on the right and he is supposed to drive on the left side of the road. It is as though English driving is a left-handed version of the right-handed American system.

I can personally attest to the seriousness of this problem. Recently my wife and I, together with another couple on an extended visit to England, decided to jointly rent a car. Usually when we drove the men would sit in the front seat, the women in the back. With our long-ingrained driving habits the world seemed totally mad. Cars, bicycles and pedestrians would dart out from nowhere and we would always be looking in the wrong direction. The car was usually filled with curses from the men and with screams and hysterical laughter from the women as we careened from one narrow escape to another. The passengers were given to sudden involuntary motions - shielding the face or slamming on non-existent brakes. The turn indicator and windshield wiper controls were also reversed from American practice and we found ourselves signaling turns with the windshield wiper - fast for a right turn, slow for a left. The whole driving situation was not particularly improved by the narrowness of English streets and the high speed of English drivers. Nor was our inner security increased by the predilection of the English for building stone walls immediately adjacent to the roads.

This paper will develop a novel solution to this problem which

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* This research was carried out in Trinity term, 1978 while the author was a Visiting Fellow at All Souls College, Oxford.

incidentally can also be used for the Englishman driving in America.

In Fig. 1 we see two triangles. They are congruent but one cannot be slid around in the plane to coincide with the other since one is, so to speak, a left-handed version of the other. A "flatlander", limited to living in the plane, could scarcely conceive how triangle A could be moved into coincidence with B, but we, as three-dimensional beings, easily understand rotating the triangle A about one of its sides and then sliding it into coincidence with the other.[1]

In an analogous way, in three dimensions we often have right- and left-handed objects - a pair of gloves, for example, or an American car compared to an English car of the same type. If we had access to a fourth dimension, one could turn a left-handed glove 180° through the fourth dimension and it would reenter the third dimension as a right-handed glove.[2] This facility would be useful in many ways. Both shoemakers and screwmakers would benefit. The former would need only right-footed lasts, the latter only right-handed taps and dies. Left-handed children could be flipped through the fourth dimension to become right-handed, since the world of tools, writing, etc., is for the most part more friendly to the right-handed. Contrariwise, right-handed baseball pitchers might choose to become southpaws. Our American driver coming to England might choose to undergo this fourth-dimensional twist which would turn his perception of England from left-handed to right-handed.

Alas, no one has found a method to rotate an object through the fourth dimension. However, equally effective would be a rotation for our American driver of all of England through the fourth dimension. This concept no doubt sounds grandiose and utterly impractical - the

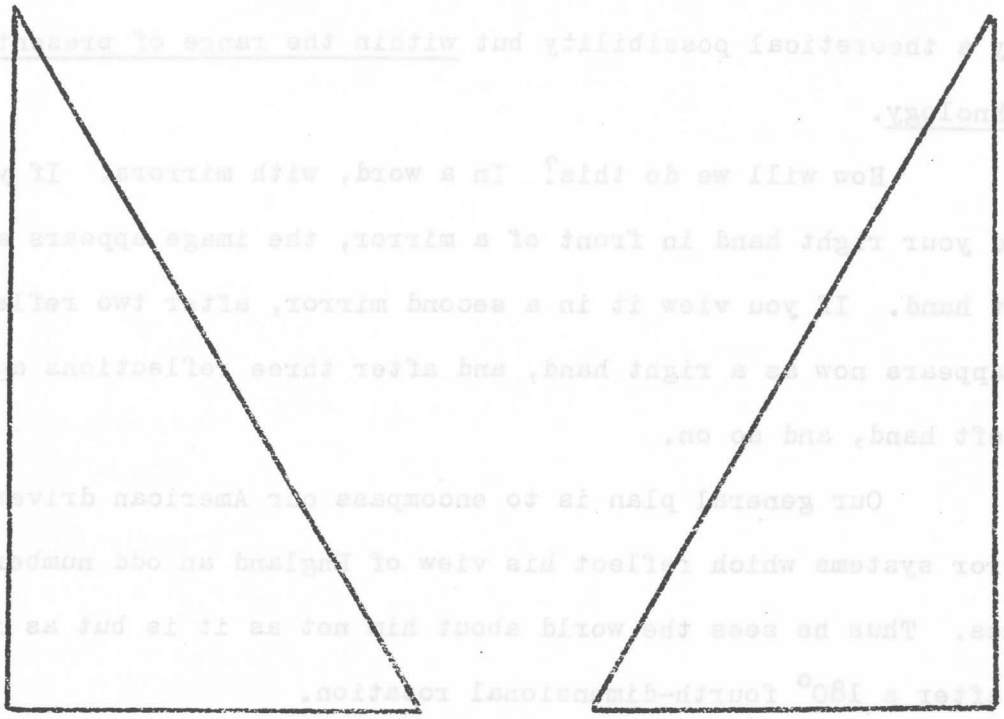

Fig..1

idle dream of a mathematician - but we will show that it is not only a theoretical possibility but within the range of present-day technology.

How will we do this? In a word, with mirrors. If you hold your right hand in front of a mirror, the image appears as a left hand. If you view it in a second mirror, after two reflections it appears now as a right hand, and after three reflections again as a left hand, and so on.

Our general plan is to encompass our American driver with mirror systems which reflect his view of England an odd number of times. Thus he sees the world about him not as it is but as it would be after a 180° fourth-dimensional rotation.

To accomplish this we have two mirror systems. The side mirror system is shown in Fig. 2, where we see the driver, from the back, sitting in his English car. There are five mirrors in the car, two on his right, two on his left, and one above his head. These serve to reflect images from the left over his head and down again so they come in from the right.[3] Similarly, light rays from the right are reflected over his head and down to come in from his left. Thus, if he turns his head to the right side of the page, he will see, by a triple reflection, an image of the object (an arrow) which is on the left of the page. In the same manner, if he looks to the left of the drawing, he will see what is on the right of the car.

To summarize, this group of five mirrors is so arranged that when he looks to his right he will see what is on his left - when he looks to his left he will see what is on his right.

Another set of mirrors provides for forward and backward vision. These are shown in Fig. 3, where we see the driver from above.

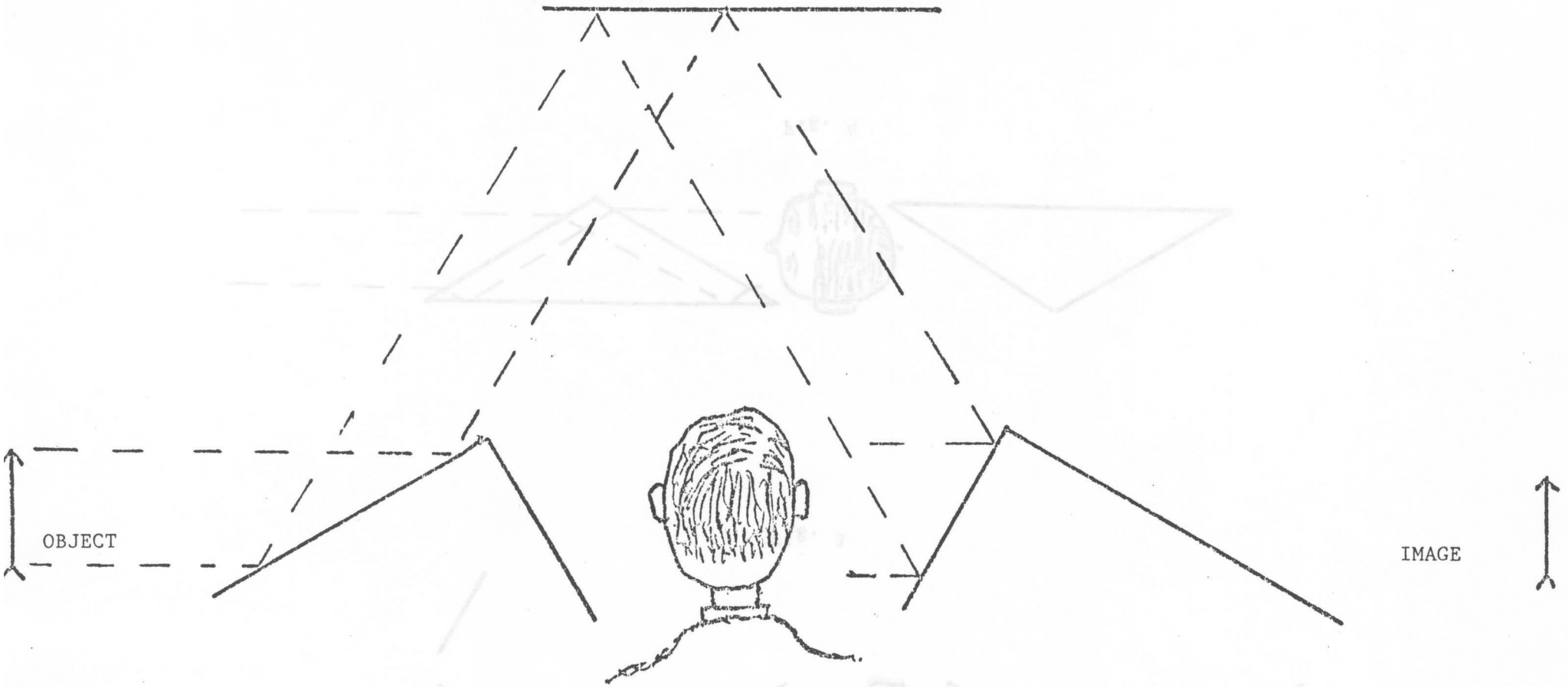

Fig. 2

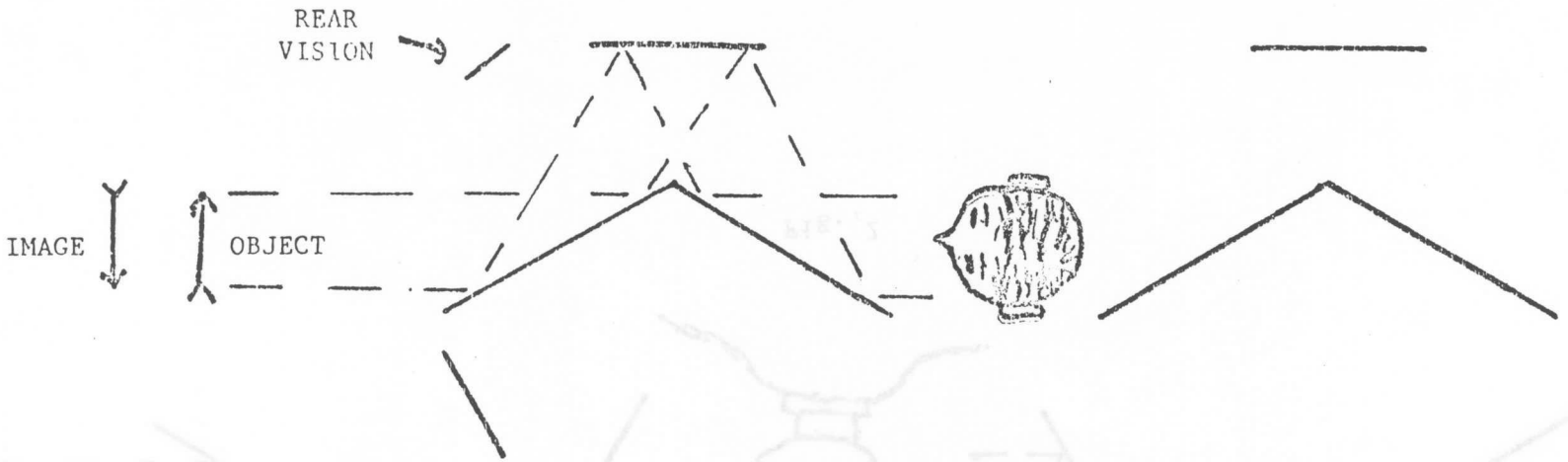

Fig. 3

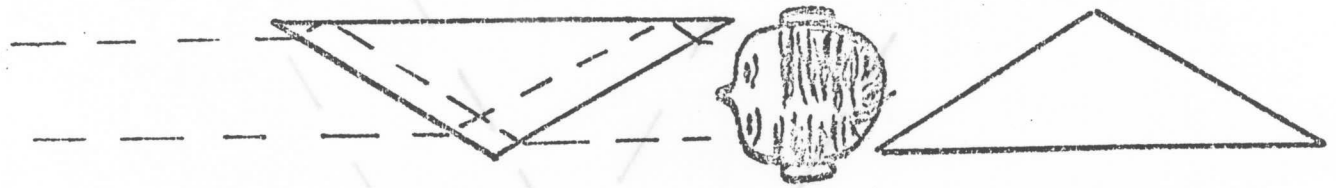

Fig. 4

For forward vision, three mirrors reflect the front visual field about a vertical axis.[4] Some light rays are indicated by letters A and B to show how the interchange of right and left takes place. The object (the usual arrow) appears to the driver as a reversed image (again somewhat farther away because of the longer path).

A second set of three mirrors accommodates vision in the backward direction. If our driver should turn his head around, perhaps in driving in reverse or possibly to look at his passengers in the back seat, he will again see a left-right reversed image.

These four mirror systems totally encompass our American driver. Wherever he looks, he sees a reversed image of England - always reflected three times. For him, England has been rotated 180° through the fourth dimension!

A further detail must be accounted for here. The rear-vision mirror in an ordinary car corresponds to one reflection - in looking through it we see words reversed and, in fact, catch a tiny glimpse of the left-handed world we have been talking about. To keep our system consistent, and to keep our American driver comfortable, we have devised a rear-vision mirror using a double reflection, as shown in Fig. 3. The driver looks up and to the right, as he would in an American car, and sees out by a double reflection through the rear window. This gives him the only glimpse he has of the real "right-hand" world, since a double reflection preserves handedness.

In Fig. 5 we see from above a car fitted with the fourth-dimensional twister. The actual car as well as the actual English road and countryside, are shown in heavy solid lines. In reality, the car is parked on the left side of the road. Another car is forward to the right and the road turns sharply to the right. The driver's perception, however, because of his mirror system which

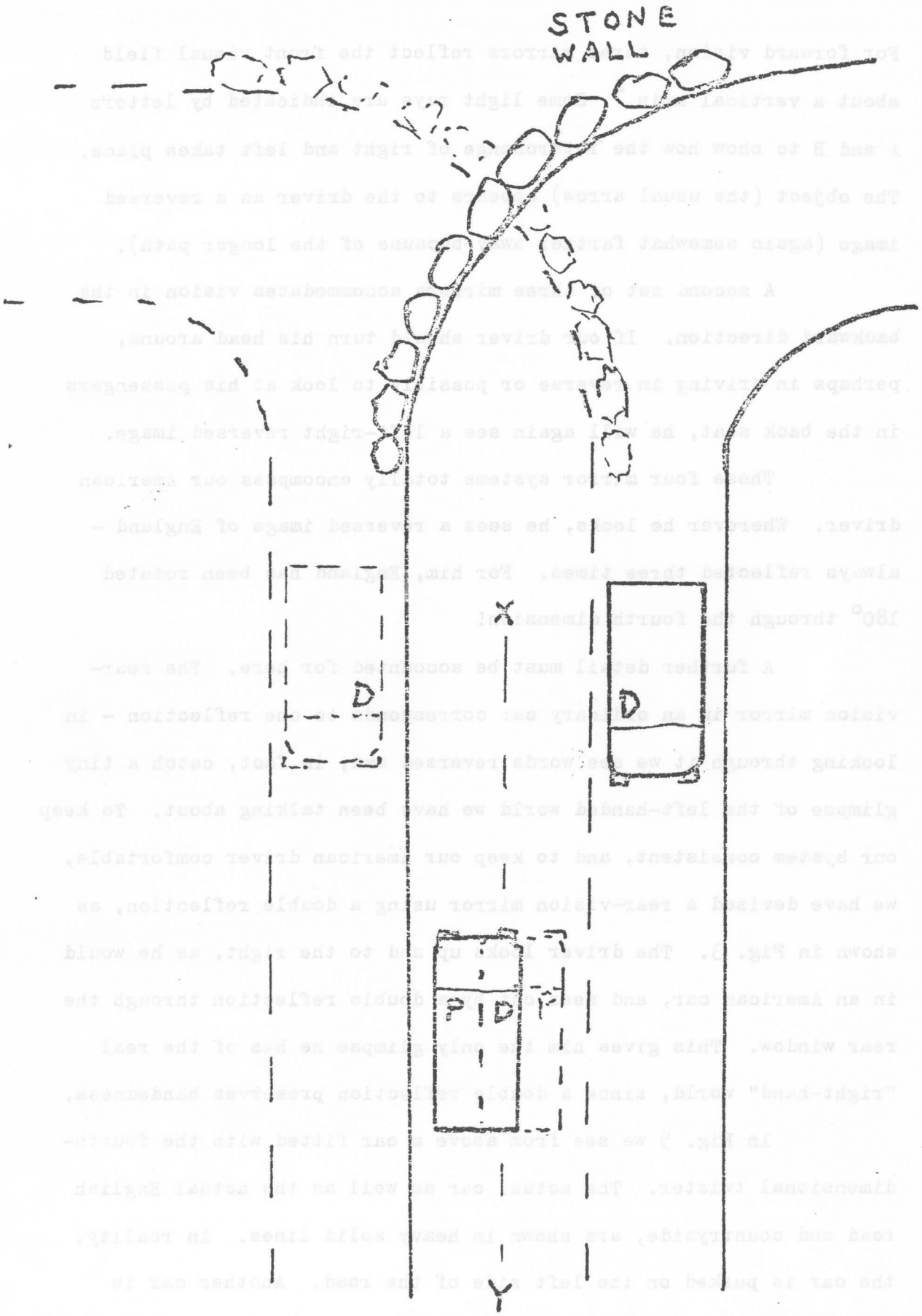

Fig. 5

reflects everything about the line XY, is that he is parked on the right side of the road, that the other car is at his left, and that the road turns sharply to the left. His perception of this situation is shown in dotted lines. Note that he even perceives his own car to have changed to an American car, and his passenger, P, on the front seat now appears to be on his right!

Entering this car may be a bit of a shock, when the entire world is reflected about a plane through the driver's seat, but after a moment our American will feel comfortable and at home, with everything as it "should be". He starts his engine and drives down the road. The road actually turns sharply to the right. In his perception of course it turns sharply to the left, so of course he turns to the left, directly into the stone wall, and is instantly killed.

This, of course, is what would have happened had we not foreseen his natural reactions to a reversed perception of the world. One must reverse not only the sensory imput but also the motor output. Fig. 6 shows an attachment to the steering wheel which reverses its operation. When turned to the right, the vehicle actually turns to the left and vice versa. This operates much as differential gears in automobiles.

With this addition our American driver will perceive a curve to the left and, in natural response, turn to the left. In fact the curve will be to the right and the mechanism will reverse his intent and turn the car to the right.

This, then, is the basic idea of the fourth-dimensional twist. There are, however, some loose ends to be dealt with. The perceptive reader may wonder about road signs. Our American driver, viewing everything through a triple reflection, sees all of the road signs

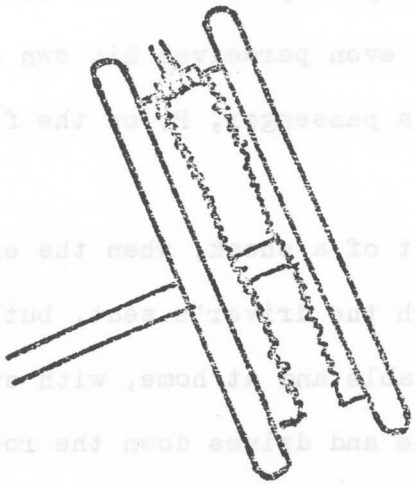

Fig. 6

in reverse, as, for example, in Fig. 7. How is he to find his way about? The answer is ridiculously simple. We have already pointed out that his rear-vision mirror gives a double reflection and hence a normal view of the real world. All he need do is back his car up to the road sign and read it through his rear-vision mirror!

A more troublesome problem is that of centrifugal force. In the situation of Fig. 5, our driver is actually turning to the right but perceives himself to be turning to the left. Centrifugal force will opt for actuality. Our driver will, surprisingly, find himself driven to the inside of the curve rather than the outside, a most uncomfortable and confusing sensation.

To solve this problem, the reversal of centrifugal force, might seem as impossible as the twist of England through the fourth dimension. After all, centrifugal force is given by the formula

$$f = m \frac{\omega^2}{R}$$

A radius R of course is always positive, ω^2 as a square is necessarily positive, and surely a mass m must be positive, so how can we arrange for the centrifugal force f to be negative? Like Columbus and the egg, the answer is very simple when given. If we immerse the mass in a liquid of higher density, it acts as though it, itself, had a negative mass. The liquid itself presses the object in the direction of acceleration!

This concept is shown in Fig. 8. Our driver is now enclosed in a scuba-diving suit within a compartment which is filled with a liquid having a specific gravity of approximately 2. Of course he would tend to rise in this liquid but he is held down firmly by his seatbelt. A snorkel provides for his breathing and altogether, with our various devices, he feels very much as though he were at home in America!

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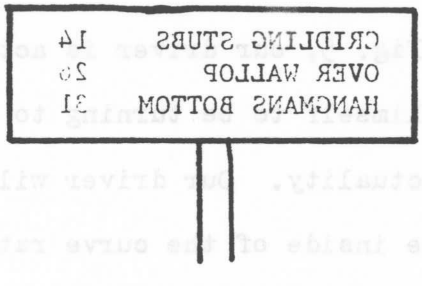

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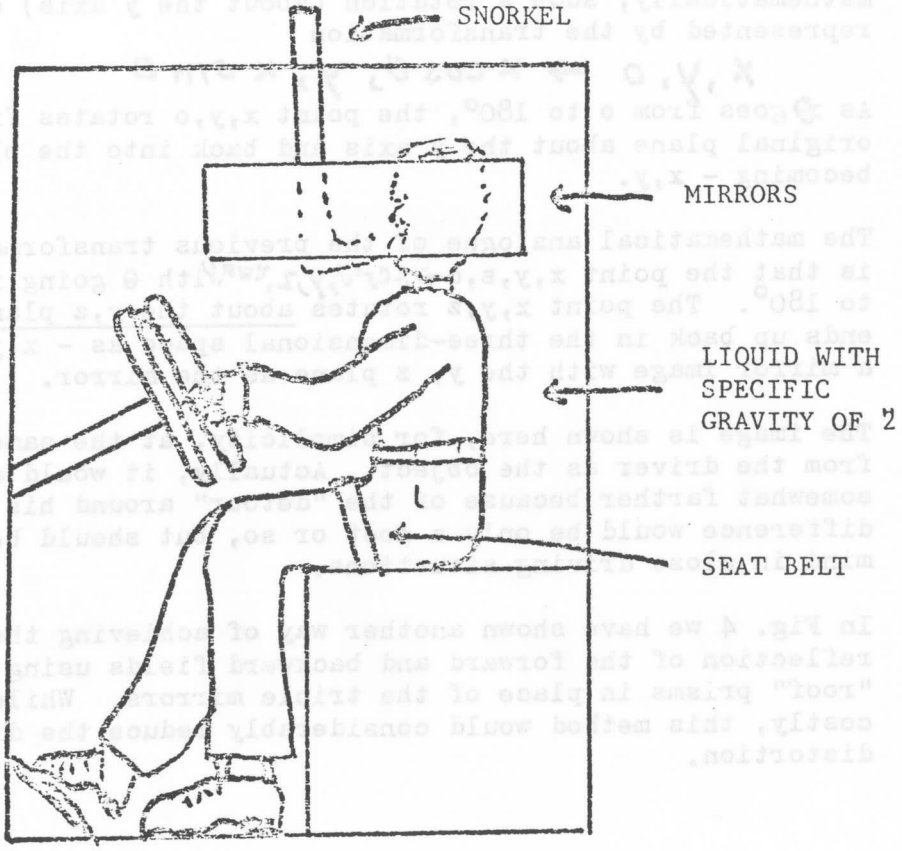
SNORKEL
MIRRORS
LIQUID WITH
SPECIFIC
GRAVITY OF 2
SEAT BELT

Fig. 8

FOOTNOTES:

1. Mathematically, such a rotation (about the y axis) can be represented by the transformation

$$x, y, 0 \rightarrow x\cos\theta, y, x\sin\theta$$

As θ goes from o to 180°, the point x,y,o rotates from the original plane about the y axis and back into the plane, becoming - x,y.

2. The mathematical analogue of the previous transformation is that the point x,y,z,o $\rightarrow$ [illegible] with θ going from 0 to 180°. The point x,y,z rotates <u>about the y,z plane</u> and ends up back in the three-dimensional space as - x,y,z,o, a mirror image with the y, z plane as the mirror.

3. The image is shown here, for simplicity, at the same distance from the driver as the object. Actually, it would appear somewhat farther because of the "detour" around his head. This difference would be only a foot or so, but should be kept in mind in close driving situations.

4. In Fig. 4 we have shown another way of achieving the desired reflection of the forward and backward fields using large "roof" prisms in place of the triple mirrors. While more costly, this method would considerably reduce the distance distortion.