

Daniel Hoek, November 2014

What may I regard as the Result of my Work?¹ [Aug. 1906]

It is almost all tied up with the concept-script. a concept construed as a function. a relation as a function of two arguments. the extension of a concept or class is not the primary thing for me. unsaturatedness both in the case of concepts and functions. the true nature of concept and function recognized.

strictly I should have begun by mentioning the judgement-stroke, the dissociation of assertoric force from the predicate Hypothetical mode of sentence composition Generality Sense and meaning

¹According to a note of the previous editors these jottings bore the date '5.VIII.06'. They are no doubt prefatory to the piece '*Einleitung in die Logik*', which is divided into sections according to the captions given here (ed.).

What I argue for

- According to Frege, it is possible to *judge* (assert) that *p* even when it is not the case that *p*.
- To judge something is just to take it to be true; and one can take something to be true even if it isn't.
- For example, it is possible to judge (assert) that Washington, D.C. is on the North Pole (even though in fact, Washington, D.C. is not on the North Pole).



What I argue against

- For Frege, judgment (assertion) is *factive* just like knowledge.
- Whenever someone judges (asserts) that *p*, *p* is true.
- Judgment (assertion) requires knowledge
- If one thinks (says) something like "Washington, D.C. is on the North Pole", one has not in fact made a judgment (assertion) at all – perhaps one has tried but failed.

What I argue against

"*Being asserted* (in this "logical" sense) is, for both Russell and Frege, something that cannot possibly attach to a proposition unless it is true."

– Elizabeth Anscombe (*An Introduction to Wittgenstein's Tractatus*, 1959)

What I argue against

"[Frege] aligns judgment with knowledge, not belief. To make a judgment is to acquire a piece of knowledge; our capacity for judgment is a capacity to arrive at knowledge"

– Thomas Ricketts, Truth in Frege (1996)

What I argue against

"[For Frege,] every assertion is an assertion of truth; every judgement, a judgement of truth."

– Robert May and Richard Heck, *Truth in Frege* (forthcoming).

What I argue against

"Judgments are always judgments of truths"

"If you say something that isn't true, then that's a failed assertion."

– Haim Gaifman (Frege seminar 2014)

What I argue against

Michael Dummett?

Dummett is on my side

"Assertions take place against the background of a custom of uttering them *with the intention* of stating something true."

[Criticizing Anscombe] "There is little enough warrant for this in Frege's published writings: I know no passage where [Frege] excludes the possibility of asserting what is in fact false."

- Dummett, Frege's Philosophy of Language (1981)

And so is Montgomery Furth

"[T]here is a puzzling aspect relating to the status of false assertions, in the sense of thoughts assertively declared to be thoughts of the True that are in fact not so. If judgment is an act, then, like other acts, it can be performed well or badly; in our attempts to designate the True our aim is sometimes inaccurate where yet we claim (wrongly yet in good faith) that we have succeeded. Of course, this much is obvious; and Frege never claims that assertion is infallible."

- Furth, Introduction to Frege's "Basic Laws"

Overview

- I. The Judgment stroke in the Begriffsschrift
- II. The Judgment stroke in the Grundgesetze
- III. Rebuttals of arguments for the opposing view
- IV. Positive evidence for my own view

The Judgment Stroke and the Content Stroke: Begriffsschrift — A — A

The Judgment Stroke and the Content Stroke: Begriffsschrift A Judgeable content







The Judgment Stroke and the Content Stroke: Begriffsschrift A "A judgment will always be expressed by means of the symbol by which stands to the left of the symbol or complex of symbols which gives the content of the judgment."



The Judgment Stroke and the Content Stroke: Begriffsschrift

[Note: if my opponents were right, there would be a whole lot of other contents that don't become judgments when preceded by a judgment stroke. E.g. the content of "Opposite magnetic poles repel each other". This would be the obvious place to mention this, and the fact that Frege doesn't do so is telling. It suggests that he thought that, unlike "|— house", expressions like

"|— 2 + 3 = 9"

do in fact express judgments: just erroneous ones.]



The Judgment Stroke and the Content Stroke: Begriffsschrift

— A

"If we *omit* the small vertical stroke at the left end of the horizontal one, the judgment will be transformed into a *mere combination of ideas* [*Vorstellungsverbindung*], of which the writer does not state whether he acknowledges [zuerkennen] it to be true or not."

Vorstellungsverbindung / beurteilbare Inhalt

"For example, let |— A stand for [bedeute] the judgment
"Opposite magnetic poles attract each other"; then — A
will not express [ausdrücken] this judgment; it is to produce in the reader merely the idea [Vorstellung] of the mutual attraction of opposite magnetic poles, say in order to derive consequences from it and to test by means of these whether the thought is correct."

Vorstellungsverbindung / beurteilbare Inhalt

"[In the Beggriffsschrift] I still combined under the expression 'judgeable content' what I have now learnt to distinguish as truth-value and thought" – Grundgesetze (1893)

"For ['Vorstellungsverbindung'], I now simply say 'Gedanke'"

"Instead of 'beurteilbare Inhalt' we can also say 'Gedanke'" – Letter to Jourdain (1910)

Vorstellungsverbindung / beurteilbare Inhalt

"We must be able to express [ausdrücken] a thought without affirming that it is true. If we want to characterise a thought as false, we must first express it without affirming it, then negate it, and affirm as true the thought thus obtained. We cannot correctly express a hypothetical connection between thoughts at all if we cannot express thoughts without affirming them, for in the hypothetical connection neither the thought appearing as antecedent nor that appearing as consequent is affirmed."

- 1910 letter to Jourdain

The Judgment stroke: Begriffsschrift

- In the Begriffsschrift, Frege emphasises assertoric force as being extrinsic to the *judgeable content* (*beurteilbarer Inhalt*) of a sentence. He notes that we can express this content without saying it's true.
- For example, consider these statements:
 - "Snow is white."
 - "If snow is white, then snow flakes must be white."
 - "Either it's Wednesday, or snow is white."
- The clause "Snow is white" occurs with the exact same *content* in all of these sentences. But only the first sentence can be used to give it assertoric force.

Some evidence that in Bgfs assertive sentences *denote* their judgeable content

- Frege uses the same symbol ('≡') for equality of judgeable content as he does for the equality of the referents of singular terms.
- He also sometimes uses "bedeuten" (as opposed to "ausdrücken") for judgeable content. (E.g. top of §5)
- "In the case [where the judgment stroke is omitted] we paraphrase [umschreiben] using the words 'the circumstance that', or 'the proposition that'"

Singular terms do not assert anything

So it sounds like in our case, we can 'paraphrase' our expression

– A

as follows:

'the proposition that opposite magnetic poles attract.'

But if that's so, it's evident that we can't be used to assert anything about the world, thus further underlining the need for a judgment stroke, if the Begriffschrift is to allow us to make assertions. Unlike a singular term, "|— A" doesn't *denote* anything: it *asserts* something.

Judgeable content in the Grundgesetze

In the Grundgesetze, expressions with judgeable content have unequivocally become singular terms:

"[T]he names " $2^2 = 4$ " and "3 > 2" denote the same truth-value, which I call for short *the True*. Likewise, for me " $3^2 = 4$ " and "1 > 4" *denote* the same truthvalue, which I call for short *the False*, precisely as 2^2 *denotes* the number 4. [...] The sense of a name of a truth-value I call a *thought*." (§2)











The Judgment Stroke: Grundgesetze

- In the Grundgesetze, Frege emphasises that there's more to an assertion than the act of referring to a truth value.
- Consider the following expressions:
 - The truth-value of "Nathan has a beard".
 - Nathan has a beard.
 - 'Nathan has a beard' = the True
- These expressions all have the exact same Bedeutung. But we can clearly never use the second one to make an assertion or express a judgment. (Singular terms don't assert anything.)

Thesis: Judgment isn't factive

arguments for and against

What we have so far

- When Frege tells us that some expressions do not become judgments when preceded by |---, he fails to remark that false statements are like this, which suggests he doesn't think they are.
- When Frege argues that we can entertain a thought without judging it to be true, he is not content to simply give a false thought, which would be the obvious example if he thought those thoughts could not be judged to be true.

The words 'Urteil' and 'Behauptung'

- 'Urteil' and 'Behauptung', just like 'Judgment' and 'Assertion' in English, are about as non-factive as it gets (akin to 'opinion', 'claim'). My opponent must hold that Frege is using these words in a very special technical sense.
- In fact, the notion of an 'Urteil' has a rich history of use as a piece of philosophical jargon in Germany, most prominently in Kant. But in that tradition the word is used non-factively (see e.g. Kremer).

Judgment as the recognition (Anerkennung) of truth

"By a judgment I understand the recognition [Anerkennung] of the truth of a thought." – Grundgesetze

"When we inwardly recognize [anerkennen] that a thought is true, we are making a judgment: when we communicate this recognition [Anerkennung], we make an assertion."

– Logic

Judgment as the recognition (Anerkennung) of truth

- Although Frege often characterises an 'Urteil' as involving the Anerkennung of truth, he is hardly consistent about this. In the Begriffschrift, he uses "Zuerkennung" (which might be best translated as "attach" or "bestow on").
- Again, the definition of an 'Urteil' as the Anerkennung of truth has a history in German philosophy. Brentano defined it that way, and he used the word 'anerkennen' interchangeably with 'annehmen', which translates roughly as 'presume' or 'suppose' (see Kremer again for details.)

Is Anerkennung really factive?

'Anerkennung' is a legalistic term which also has informal uses, meaning something like 'official admission, recognition, appreciation or acknowledgement'.

'Die Internationale Anerkennung des Staates Palästina'

'Die Soldaten verdienen mehr Anerkennung',

'Vaterschaftsanerkennung',

'Die Ehe als rechtsgültig anerkennen'

'Ein Anspruch anerkennen'

Is Anerkennung really factive?

Although Ricketts is perhaps right that in ordinary usage, the word 'anerkennen' sometimes implicates truth, this is by no means an entailment. For example, the following sentences are fine:

"Die gefälschten Dokumente sind als offizielle Dokumente anerkannt geworden."

"In der Vergangenheit habe ich diesen Gedanke irrtümlicherweise als wahr anerkannt."

"Italien hat Nimmerland offiziell als Staat anerkannt, aber Nimmerland existiert natürlich gar nicht."

(Thanks to Max Barkhausen, Vera Flocke and Milena Bartholain for vetting these examples.)

Is Anerkennung really factive?

- In short Anerkennung isn't really factive at all, and it has a history of non-factive use in philosophy.
- If my opponent wishes to maintain that Fregean judgment is factive, he must maintain that Frege uses *both* the word 'Urteil' *and* the word 'Anerkennung' in a non-standard way
 - *both* from the perspective of the natural language use *and* from the perspective of established philosophical usage by Frege's contemporaries.
 - without signalling this anywhere.

Recap

- In the Begriffsschrift, Frege emphasises assertoric force as being extrinsic to the *judgeable content* (*beurteilbarer Inhalt*) of a sentence. He notes that we can express this content without saying it's true.
- For example, consider these statements:
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Recap

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Recap

Judgment is not factive

- If it were, Frege is guilty of some peculiar sins of omission
- When he argues that not all contents can become judgments, he fails to mention false thoughts.
- And again when he argues that we can entertain a thought without judging it to be true.
- Urteil, Behauptung and Anerkennung are all non-factive, both in German and philosophy German
- (Frege characterises judgment on empirical matters as risky.)

Judgment is factive

- Judgment involves the recognition (Anerkennung) of truth.
- (Frege takes care to place the judgment stroke only in front of true statements)
- (Providing a fundamental link between truth and judgment may be the only way to honour the special role of the truth in Frege's semantics)

"Frege takes great care to use the judgment stroke only in front of true Begriffsschrift statements".

- (E.g. in the appendix to Vol. II of the Grundgesetze, Frege omits the judgment stroke in front of Basic Law V and some of its consequences.)
- This suggestion doesn't tell against my interpretation. The formulae of Begriffsschrift Frege prefaces with an judgment stroke are *assertions by Frege*. We all knew that Frege takes great care to assert only true things.
- Aside from that, the statement is not quite true. In the expository parts of his works, Frege sometimes uses an judgment stroke without regard for whether the statement that follows it is true.

Introduction of the Conditional Stroke

"Let A and B denote judgeable contents, then there are the following four possibilities:

A is affirmed and B is affirmed A is affirmed and B is denied A is denied and B is affirmed A is denied and B is denied

Now



stands for the judgment that the third of these possibilities does not take place, but one of the others does.

Introduction of the Conditional Stroke

It follows from what Frege says here that in the case where A is denied (false) and B is affirmed (true), too,



stands for the judgment that it is not the case that A is false and B is true.

That would have to be an example of an incorrect judgment.

Introduction of the Conditional Stroke

Let A and B denote judgeable contents, then

denotes the **circumstance** that the possibility that *A* is false and *B* is true does not obtain. For example, if Δ is the circumstance that the Moon is in quadrature, and Γ the circumstance that it appears as a semicircle, then

denotes the **judgment** that if the Moon is in quadrature, then it appears as a semicircle.

Introduction of the Negation Stroke

^{57.} If a small vertical stroke is attached to the underside of the content stroke, then this is intended to express the circumstance that *the content does not obtain*. Thus, for example,

$\vdash - A$

means 'A does not obtain'. I call this small vertical stroke the *negation* stroke. The part of the horizontal stroke to the right of the negation stroke is the content stroke of A, the part to the left of the negation stroke, on the other hand, is the content stroke of the negation of A.

Frege's doctrine of inference (Schliessung)

- In the Begriffsschrift and the Grundgesetze, arguments are conducted by presenting both the premises and conclusion as judgments.
- This has to do with Frege's doctrine that good inferences are *sound*: "We can only infer something from true propositions", and therefore "recognition of the truth of the premises is required" to go ahead with the inference. (Letter to Dingler 31.1.1917)
- But if recognition isn't factive, our recognition of the truth of the premises isn't sufficient to guarantee their actual truth.

Frege's doctrine of inference (Schliessung)

- In the Begriffsschrift and the Grundgesetze, arguments are conducted by presenting the premises and conclusion as judgments.
- His justification for doing things this way is as follows: "we can only infer something from true propositions", and therefore "recognition of the truth of the premises is required" to go ahead with the inference. (Letter to Dingler 31.1.1917). Hence the premises must be prefixed by judgment strokes.
- But if judgment isn't factive, our judgment of the truth of the premises isn't sufficient to guarantee their actual truth.

Frege's doctrine of inference (Schliessung)

- Sure, Frege's assertion that the premises are true is no guarantee that they are indeed true.
- Nor is Frege's use of the contraposition symbol 'X' any guarantee that the argument in which it occurs is in fact an instance of contraposition.
- Nevertheless, the correctness (Richtichkeit) of the inference, and our grounds for believing the conclusion, depend on the premises being true, and on the form of the argument indeed being instance of contraposition. That it why the notation should signal that the author attributes those properties to the argument, even if these attributions do not guarantee that the argument actually possesses them.

Draft of a letter to Jourdain

"Whoever understands a sentence [Satz] uttered with assertoric force [behauptender Kraft] adds to it his recognition of the truth. If a sentence uttered with assertoric force expresses a false thought, then it is logically useless and strictly speaking, incomprehensible. A sentence uttered without assertoric force can be logically useful even if it expresses a false thought, e.g. as part (antecedent) of another sentence."

Draft of a letter to Jourdain

- A good deal of speculation is needed to get the reading of this passage my opponent would need.
- Frege never mailed this letter. He instead sent Jourdain a revised version, from which he omitted the whole discussion of inferences and judgements.
- In later publications by Frege, such as *The Thought*, we no longer see any hint of this kind of talk.

Switching (Kremer)

Frege is prone to switch between *Urteilen* and *Anerkennen* and manifestly non-factive expressions like *fürwahrhalten* in what appear to be simply reiterations of his views.

- "These two [truth values] are recognized, if only implicitly, by everybody who judges at all, who holds something true—and so even by a skeptic." *Sinn und Bedeutung*
- "truth is different from fürwahrhalten" ... "truth is independent of Anerkennung" *Grundgesetze*
- "How does a thought act? By being grasped and held true."
 ... "our actions are usually led up to by thinking and judging."
 Der Gedanke

Judgment is Risky (Kremer)

- "A sense impression is not in itself a judgment, but becomes important in that it can lead us to judging. Then mistakes can occur, sense-illusions." ... "there are at our disposal a diversity of means for correcting the judgment (*Urteils*) gained from the first impression." – *Sources of Knowledge in M. and N.S.*
- "the probability is nevertheless in many cases hard to distinguish from certainty, so we can venture to judge about things in the external world. And we must make this venture even at the risk of error if we do not want to fall into far greater dangers." – *The Thought*

Tidbits

For example, let F be the property of being a heap of beans; let f be the procedure of removing one bean from a heap of beans; so that f(a, b) means the circumstance that b contains all beans of the heap a except one and does not contain anything else. Then by means of our proposition we would arrive at the result that a single bean, or even none at all, is a heap of beans if the property of being a heap of beans is hereditary in the f-sequence. This is not the case in general, however, since there are certain z for which F(z) cannot become a judgment on account of the indeterminateness of the notion "heap".

(Van Heijenoort (ed.), Begriffsschrift)

Tidbits

"If someone wanted to contradict the statement that what is true is true independent of our recognition, he would **by his very assertion** contradict what he asserted, similar to the way in which a Cretan who said that all Cretans lie would." – *Logic*

Tidbits

"We should not wish to deny sense to a command, but this sense is not such that the question of truth could arise for it. Therefore I shall not call the sense of a command a thought. Sentences expressing wishes or requests are ruled out in the same way. Only those sentences in which impart or assert something [in denen wir etwas mitteilen oder behaupten] come into question." – The Thought

Tidbits

"The hearer does not have to take the speaker's stance [on a given assertion]; not that he has to reject it either. He can simply refrain from making a judgment." – *Logical Doctrines*

Time to Tally!

Judgment is not factive

- If it were, Frege is guilty of some peculiar sins of omission in both the Begriffsschrift and the Grundgesetze
- Urteil, Behauptung and Anerkennung are all non-factive in both German and philosophy German
- Frege sometimes brands a class of expressions as judgments, some of which are false
- Frege switches between urteilen and fürwahrhalten in a way that suggests he regards them as equivalent
- Judgment is risky
- Tidbits

Judgment is factive

- Judgment involves the recognition (Anerkennang) of truth.
- Frege tries to prefit the judgment stroke only to truths
- Fregean doctrine of inference
- Three words from an ambiguous passage from an unmailed letter to Jourdain

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