Hyperintensionality and Propositional Mereology

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> "The conclusions we draw from such a definition extend our knowledge, and ought therefore, on Kant's view, to be regarded as synthetic; and yet they can be proved by purely logical means, and are thus analytic. The truth is that they are contained in the definitions, but as plants are contained in their seeds, not as beams are contained in a house."
>
> — Frege, *Foundations of Arithmetic*, §88

While the clauses of a conjunctive definition are contained in the definition "like beams in a house," other entailments are like "plants in a seed." The beams are part of the house, but a plant is not part of its seed. So Frege can be seen here as hinting at the distinction between entailments that are ***part*** of the entailing proposition and those that are not part of it. That distinction is reflected in natural language. Suppose we are considering watching *Brazil*, and Mora says:

1) *Brazil* is an amazing movie with Frances McDormand.

We watch the movie and while it is wonderful, McDormand is not in it. Then one could say to Mora:

2) (a.) Well, part of what you said was true: (b.) that was a great movie.

By contrast, the following remark makes very little sense:

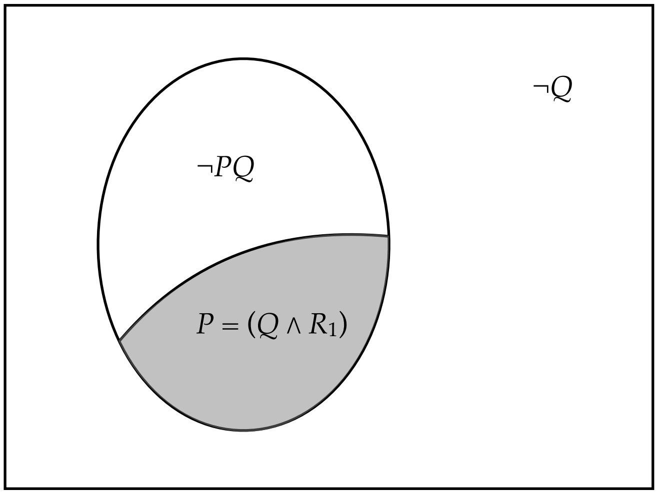
3) (a.) Well, part of what you said was true: (b.) either Frances McDormand was in that movie, or it was directed by Terry Gilliam.

Both (2b) and (3a) are *entailed* by what Mora said (1), but only (2b) is *part* of what she said. (Similarly sensitive locutions that can be substituted for (2/3a) include "What you said was *partly* true," "At least you got *something* right," "Some of what you said was true.") The notion of propositional parthood is close to that of ***analytic entailment*** (Parry 1933, Angell 1977, Gemes 1997, Fine 2016/17).

Last week we saw that, Jaeger collapsed the distinction between entailments and parts in his treatment of logical subtraction. One natural starting point for trying to make progress on the problems he ran into would be to reinstate that distinction, and that's what we will attempt to do this week. On our way to a theory of propositional parts, we will introduce two other important semantic concepts:

- ***Subject matters*** (known in linguistics as ***questions*** or ***issues***).
- ***Truthmakers*** (known in linguistics as ***situations*** or ***states***).

Just as intensionally equivalent conjunctions may have different conjuncts, so truth-conditionally equivalent propositions can intuitively have different parts. Both subject matters and truthmakers will be helpful in drawing appropriate the appropriate hyperintensional distinctions.



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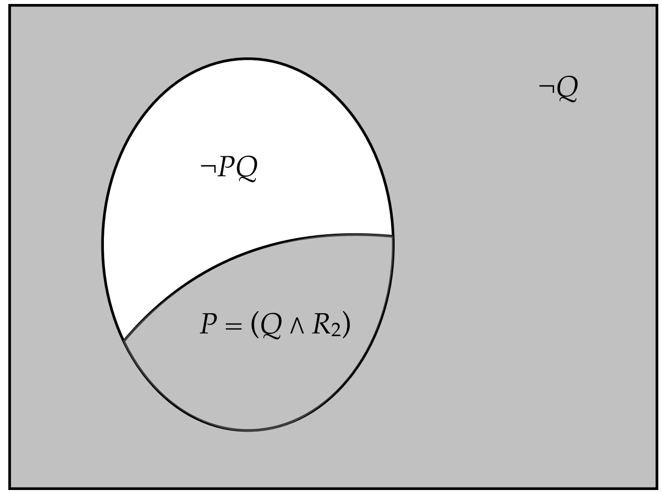

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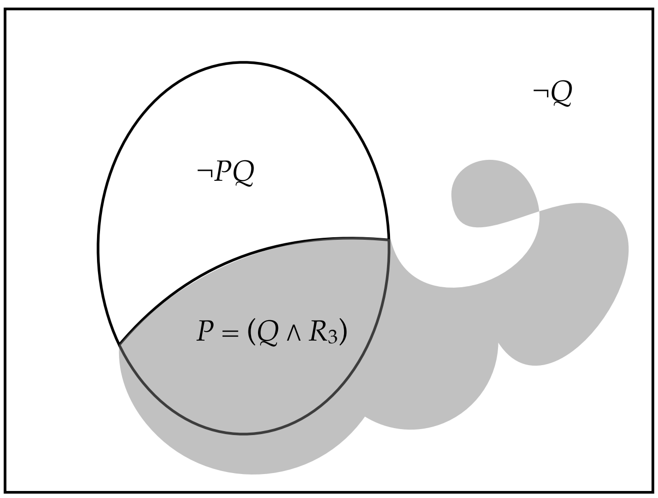

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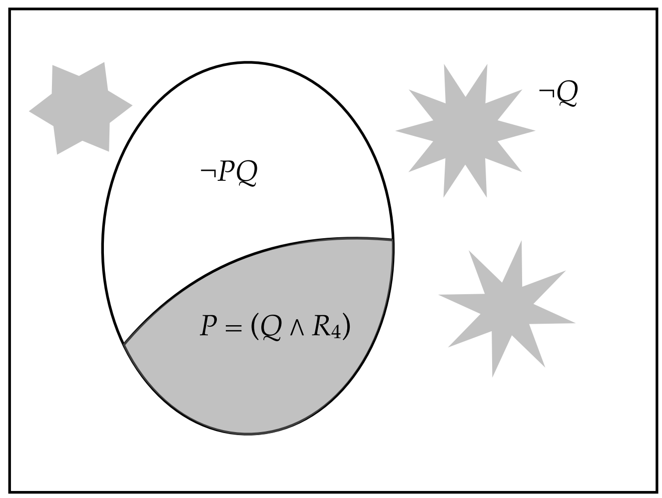

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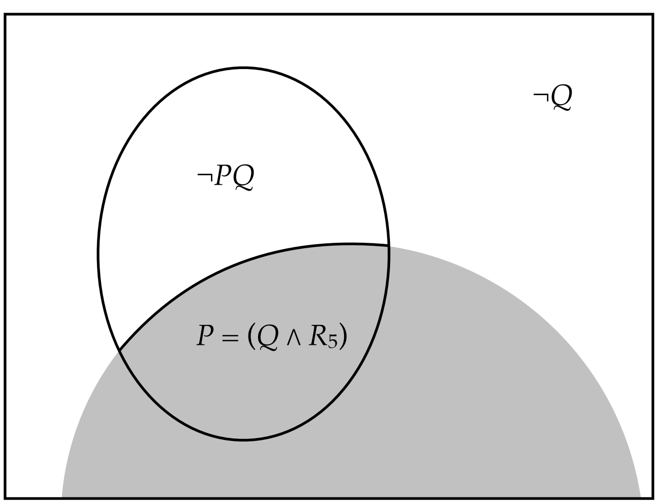

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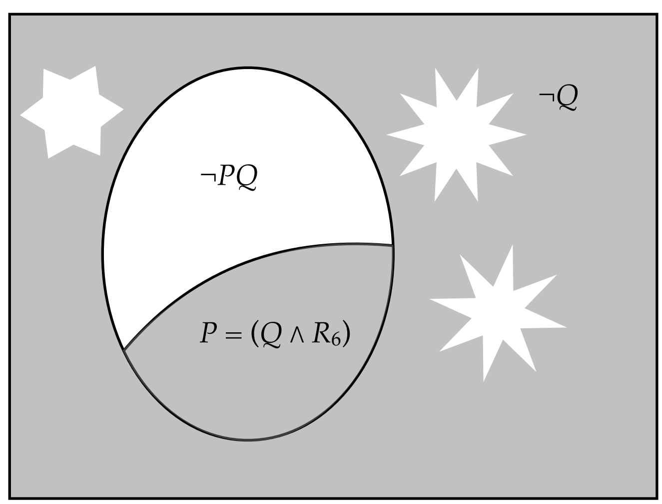


VI.

Recap: Jaeger's Problem

In the last class, we accomplished the following:

- We observed that, in many cases, it is cumbersome or even impossible to reformulate the claim intuitively expressed by $P - Q$ or its English counterpart "P, except maybe not Q".
- We explored a host of potential linguistic and philosophical applications of logical subtraction, including not only applications in the philosophy of language, but also the philosophy of
- We saw that intuitively speaking, $P - Q$ makes sense for some choices of P and Q but not for others, and speculated about the conditions under which Q is extricable from P.
- We saw that Jaeger's conditions for subtraction leave open a wide range of possible remainders $P - Q$ for any given P and Q.

This final observation, illustrated by the diagrams on the left, gives rise to what we'll call *Jaeger's problem*. This problem will be our starting point today.

In each of the six diagrams, the shaded regions represent candidates for the remainder $R = P - Q$. That is to say, they represent different propositions R such that P is truth-conditionally equivalent to $(Q \wedge R)$. In particular, $R_1 = P$ and $R_2 = (Q \supset P) = (\neg Q \vee P)$. All of the candidate Rs are truth-conditionally equivalent to some proposition of the form $((Q \vee S) \supset P) = ((\neg Q \wedge \neg S) \vee P)$, where S can be any proposition that includes (i.e. is entailed by) the unshaded region outside the oval. The fact that the constraint $P = (Q \wedge R)$ determines no unique remainder R raises a problem for the notion of logical subtraction. The problem can be formulated in two closely related ways.

Underdetermination Problem. Given the wide variety of options available, it is reasonable to wonder whether the propositions P and Q radically underdetermine the remainder $(P - Q)$. Insofar as that leaves the expression $(P - Q)$ radically ambiguous, this is problematic both for the prospect of putting the notion of logical subtraction to fruitful use in conceptual analysis or in semantics.

Inverse Problem. One of our initial characterisations of the notion of logical subtraction was as the inverse of conjunction, so we would like to vindicate the following schema as much as possible:

4. $(Q \wedge R) - Q \quad = \quad R$

But if propositions are individuated by their truth-conditions, counterexamples to (4) will be rampant. For instance, suppose that (4) holds for R_3, so that $(Q \wedge R_3) - Q = P - Q = R_3$. Then every other candidate R_i will be a counterexample to (4), since $(Q \wedge R_i) - Q \ = \ P - Q \ = \ R_3 \ \neq \ R_i$.

Subject Matters

There are two broad strategies for responding to Jaeger's problem:

- *The hyperintensional strategy*. Propositions are not individuated by their truth-conditions, and the underdetermination is to be mitigated by attending to the hyperintensional features of propositions P and Q. Jaeger's problem is to be addressed by replacing intensional notions of *entailment* and *independence* with hyperintensional notions like *parthood* and *orthogonality*.
- *The context-dependent strategy*. The value of the expression "$P - Q$" depends on features of the context in which it occurs. (4) is not generally valid, but it does hold relative to suitable contexts (for instance contexts in which R is *relevant*).

Broadly speaking, Yablo follows the hyperintensional strategy, while I follow the context-dependent strategy. Fine and Humberstone arguably follow a mixed strategy. (In practice, any strategy ends up being somewhat mixed: context often informs the natural hyperintensional interpretation of the sentence, while the wording of the sentence can affect relevant aspects of the context.)

The Lewisian notion of subject matter is involved in both these strategies. On the one hand, subject matter is plausibly a hyperintensional aspect of meaning. On the other hand, subject matter is an aspect of contexts. In conversation, there is always something we are talking *about*. A contribution to the conversation is directly relevant if it is about that subject matter, and irrelevant otherwise. In linguistics, the subject matter we are talking about is called the ***Question Under Discussion*** or ***QUD.***

Lewis on Subject Matter

Lewis considers three different characterisations of subject matter:

A) Subject matters as intensionally individuated parts of the world.
   - The subject matter *The eighteenth century* is a continuous spatiotemporal chunk of the world.
   - The subject matter *Styrofoam*, consists of the sum total of all the styrofoam in the world — it picks out a different discontinuous part of the universe at each world

B) Subject matters as equivalence relations (an ***equivalence relation*** is a transitive, symmetric, reflexive relation — for example, having the same height is an equivalence relations).
   - The subject matter *The eighteenth century* is the relation $\sim_E$ such that $w \sim_E v$ if and only if w and v agree on everything that took place in the eighteenth century (but they can differ on what happened at any other time).
   - The subject matter *The number of stars* is the relation $\sim_N$ such that $w \sim_N v$ if and only if w and v contain the same number of stars.

- In general, the subject matter S corresponds to the relation $\sim_S$ such that $w \sim_S v$ if and only if w and v agree on all the facts about S.

C) Subject matters as partitions of logical space. A ***partition*** of logical space is a set of mutually exclusive non-empty sets of possible worlds that jointly cover all of logical space. For instance

$$\{ \{ w : \text{penguins can fly at } w \}, \{ w : \text{penguins can't fly at } w \} \}$$

- The subject matter S corresponds to the partition $\{ \{ w : v \sim_S w \} : w \in \Omega \}$

Lewis prefers (B) over (A), because it's more general: "Maybe an ingenious ontologist could devise a theory saying that each world has its *nos-part,* as we may call it, such that the nos-parts of two worlds are exact duplicates iff those two worlds have equally many stars. Maybe-and maybe not. We shouldn't rely on it." (p. 12). (B) and (C) both come to the same thing, in that every equivalence relation on worlds corresponds to a partition and vice versa.

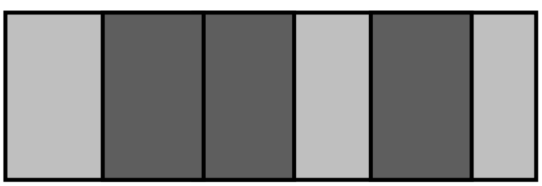
Relations between subject matters and intensional propositions (sets of worlds):

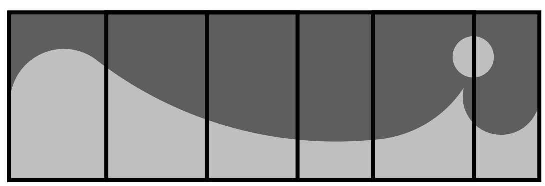
i) An intensional proposition P is ***(wholly) about*** S if and only if P has the same truth value at w and v whenever $w \sim_S v$.
   - Equivalently, P is about S if and only if P is a union of S-cells.

ii) An intensional proposition P has ***a bearing*** on S iff there is an S-cell that P rules out.

iii) An intensional proposition P has ***no bearing*** on S iff P intersects every S-cell.

iv) An intensional proposition P is ***orthogonal*** to S iff both P and $\neg P$ intersect every S-cell.

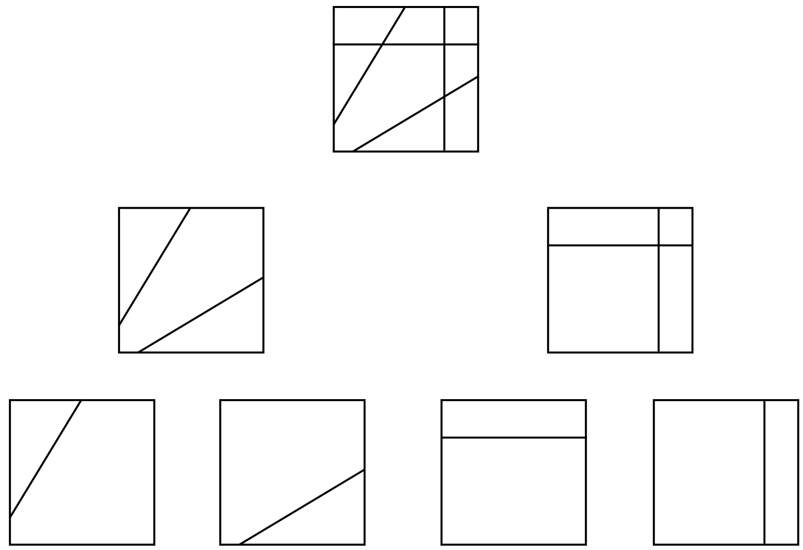




Let S *be the partition represented by the black lines. The colouring represents two propositions: light grey for worlds where it's true. One of these proposition is about* S*, the other is not.*

Relations between subject matters:

v) A subject matter S ***contains*** a subject matter T, or T is ***part of*** S if and only if S is a fine-graining of T, that is if and only if S makes every distinction between worlds that T makes, and possibly more besides. (Linguists call this question *entailment*).
   - Equivalently, S *contains* T if and only if every T-cell is a union of S-cells,
   - S *contains* T if and only if every S-cell entails a T-cell,
   - S *contains* T if and only if every proposition about T is also about S,
   - S *contains* T if and only if $\sim_S$ entails $\sim_T$.



Subject matter parts and subject matter conjunction

vi) The ***conjunction*** or ***fusion*** ST of S and T is the smallest subject matter containing both.

- Equivalently, $ST = \{ \{ s \cap t \} : s \in S, t \in T\} \setminus \{\emptyset\}$

vii) The ***intersection*** $S \vee T$ of two subject matters S and T is their greatest common part.

viii) Two subject matters are ***disjoint*** if their intersection is the trivial subject matter $\{ \Omega \}$. They overlap iff they are not disjoint.

ix) Two subject matters S and T are ***orthogonal*** iff every S-cell is orthogonal to T .

Since fusion and intersection are always well-defined, subject matters form a lattice. But its not a very well-behaved lattice. In particular partition lattices are *non-distributive*: that is, in general $S \vee (T \wedge U)$ does not equal $(S \vee T) \wedge (S \vee U)$ and $S \wedge (T \vee U)$ does not equal $(S \wedge T) \vee (S \wedge U)$. And we cannot define a natural notion of complementation or negation for subject matters.

Alternatives

Alternative sets-of-sets-of-worlds charactarisations of subject matter:

- One can drop the requirement that the cells of the subject matter be mutually exclusive.
  - The question *Where can I buy an Italian newspaper* is answered by saying *At the hotel* and also by saying *At the newsstand*. At least in the right context, these seem like complete answers to the question, but they are compatible with each other.
  - Yablo argues that the subject matter *Joe's approximate height* has overlapping cells: the answers *around five foot seven* and *around five foot eight* are consistent.

- One can also drop the requirement that the cells of the subject matter be *exhaustive*.
  - One motivation for this are questions with presuppositions, such as *How old is the King of France*, *Who gave Rob a black eye* and *What is your favourite colour*.
  - According to inquisitive semantics, disjunctions are non-exhaustive questions: *Milica is either in the park or at the restaurant*.
  - In Yablo's and Fine's framework, propositions do not just have an overall subject matter but also a *positive* and *negative* subject matter. The positive subject matter is only defined where the proposition is true, and the negative subject matter is only defined where the proposition is false.

Beyond possible worlds:

- Lewis' initial conception of a subject matter as a part of the world.
- In Fine's formalism, subject matters are defined as *states*. It is left somewhat indeterminate exactly what a state is. But Fine is clear they are not supposed to be sets of worlds. (In particular, the overall subject matters of bilateral propositions are always impossible states, and there is more than one of those).

From Subject Matter to Truthmakers

Intuitively, the subject matter of a proposition, what a proposition is *about*, is an aspect of its meaning. Lewis tries to define the *minimal subject matter* of an intensional proposition, but runs into the problem that, for any proposition p, its subject matter ends up being binary subject matter {p, ¬p}: thus the only propositions about the same subject matter is its negation. In fact, subject matter seems to be a hyperintensional aspect of meaning. Consider for instance the following pairs of sentences:

4a) All ravens are black.
4b) All non-black things are non-ravens.

5a) Either penguins can fly or penguins can't fly.
5b) Whenever it snows, it snows.

6a) England can avoid war with France.
6b) England can avoid war and also nuclear war with France.

7a) Either Amir came to the party without Andres, or Amir and Andres both came to the party.
7b) Amir came to the party.

In each case, the two sentences (a) and (b) are logically equivalent, but intuitively they are not about the same subject matters.

The simplest way to incorporate subject matters into our conception of propositions, is to say that a proposition is jointly individuated by its subject matter and its truth conditions. Presumably the truth-conditions must be appropriately related to the subject matter, and we shall assume that propositions are *wholly about* their subject matters in Lewis' sense. This allows us to define:

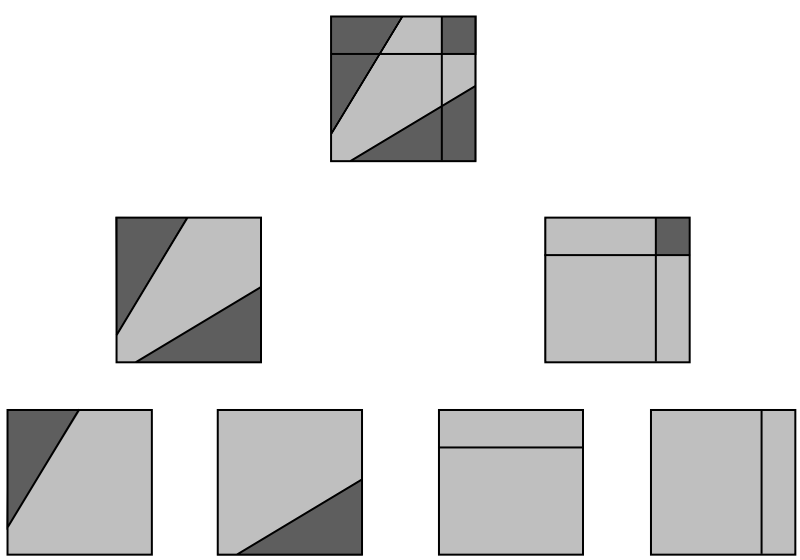
> A ***subject-specific proposition*** is an ordered pair $\langle S, P \rangle$ of a subject matter S and a set of cells P, also written P^S. Then P^S is ***true*** if and only if the actual world is in some P-cell, and ***false*** otherwise. Moreover, P^S is ***true at a world*** w iff $w \in \bigcup P$ and ***false at*** w otherwise.

Yablo calls these sorts of propositions *directed propositions*. In other work I have refer to them as *question-specific propositions* or ***quizpositions*** for short. And I'm now so used to it now, that I'll just adopt that terminology for this class as well.

One can think of a quizposition P^S as a region in the space of complete answers to the question S, rather than a region in the space of possible worlds. So the cells of the subject matters play the role of small possible worlds: possible ways that a specific aspect of the world might be, rather than the world as a whole. That is the basic idea of truthmakers:

> A cell $s \in S$ ***makes*** P^S ***true*** iff $s \in P$, and $s \in S$ ***makes*** P^S ***false*** otherwise. Correspondingly, the cells in P are the ***truthmakers*** or *verifiers* of P^S, and the cells in $S \setminus P$ are the ***falsemakers*** or *falsifiers*.

(For reasons I'll touch on, some intensional propositions that are not in S may also end up counting as truth- or falsemakers for P^S. But for now we don't need to be worried about that.)



Quizposition Parthood and Quizposition Conjunction

Parthood and Conjunction

Here is a a simple characterisation of the Boolean operators for quizpositions:

$\neg P^S = (S \setminus P)^S$

$P^S \wedge Q^T = (PQ)^{ST}$

$P^S \vee Q^T = \neg(\neg P^S \vee \neg Q^T)$

Here PQ is defined as you would expect: $\{ \{ p \cap q \} : p \in P, q \in Q\} \setminus \{\varnothing\}$. A quizposition conjunction makes just enough distinctions between possible worlds to make every distinction that its conjuncts make, and rules out just enough possibilities to rule out every possibility that its conjuncts rule out.

Quizposition parthood is the relation that quizposition conjuncts bear to their conjunction: a quizposition is part of another if it makes fewer distinctions and rules out fewer possibilities.

> A quizposition P^S ***contains*** a quizposition Q^T, or Q^T is ***part of*** P^S, if and only if Q contains R and A entails B (that is, $\bigcup A \subseteq \bigcup B$).

As in the case of questions, one quizposition contains another just in case the conjunction is equal to the whole. That is to say, P^S contains Q^T if and only if $PQ^{ST} = P^S$.

Alternative Accounts of Truthmaking

- The most important simplification forced by the use of quizpositions that it does not allow us to give the special treatment of *disjunction* that is characteristic of the truthmaker based accounts by Van Fraassen, Fine and inquisitive semantics.
  - On those accounts, the (exact) truthmakers for $P^S \vee Q^T$ are just the truthmakers of P^S and the truth-makers of Q^T.
    - Thus the truthmakers don't have to form a partition. For example, the truthmakers of "Goats eat cans or penguins can fly" will overlap.
    - Even if we close truthmakers under conjunction, as Fine proposes, there still won't be a truthmaker to cover the state that *Goats eat cans and penguins can fly*.
  - On the present treatment, the truthmakers for $P^S \vee Q^T$ are conjunctions of S- and T-cells.
- *Unilateral* conceptions of truthmaker propositions dispose of the notion of falsemakers, instead identifying a proposition with a set truth-makers. Such a characterisation is adopted in the inquisitive semantics literature, and Fine considers it too.
  - This approach gives rise to a problem about negation: ordinarily, the truthmakers of ¬P are defined as the false-makers of P. But on this view, there are no falsemakers.
- For Kit Fine, truth makers are not just sets of worlds, but *states* that he takes to be individuated

hyperintensionally, and which stand in mereological relations to one another. (But his formalism is compatible with the interpretation of states as sets of worlds, interpreting $\sqsubseteq$ as entailment.)

- Yablo and Fine distinguish the *overall subject matter* S of P^S from its *subject matter* P and *subject anti-matter* (P\S). Fine calls these *bilateral subject matter*, *positive subject matter* and *negative subject matter* respectively.

Other Notions of Parthood

Yablo and Fine define propositional parthood as follows:

A directed proposition *P* ***contains*** the directed proposition *Q* if and only if:

i) Every truthmaker for *P* entails (contains) a truthmaker for *Q*
ii) Every truthmaker for *Q* is entailed by (part of) a truthmaker for *P*
iii) Every falsemaker for *Q* is a falsemaker for *P*

The core idea here is the same: *P* contains *Q* just in case $(P \wedge Q) = Q$. The differences are mostly informed by the differences in the definition of disjunction alluded to earlier. This affects the definition of a conjunction too, since the falsemakers of $(P \wedge Q)$ are the truthmakers of $(\neg P \wedge \neg Q)$; hence the definition of parthood is affected as well.

Let me also mention Yablo's notion of a *part about a subject matter*. This is related to the notion of a *maximal* part: if T is part of S, then the maximal part of P^S about T is its strongest part about T.

Back to Logical Subtraction

We have now defined hyperintensional analogues to the notions of conjunction, entailment and logical independence that Jaeger used: we can now replace them with quizposition conjunction, parthood and disjointness. Making these substitution gets us the following characterisation of $R^U = P^S - Q^T$:

O. P^S contains Q^T
I. P^S contains R^U ("What is left over will be a part of the original whole.")
II. $Q^T \wedge R^U$ contains P^S ("The whole is equal to the sum of its parts.")
III. U contains no part of T ("What is subtracted cannot be a part of what is left over.")
IV. T contains no part of U ("What is left over cannot be part of what was subtracted.")

Does this resolve the underdetermination problem?