## Questions in Action

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#### Abstract

Choices confront agents with questions. Lost in a dark forest and coming to a fork in the road, you wonder Which path will get me out of here? The choice of how many eggs to buy at the supermarket raises the question How many eggs go into a spaghetti carbonara for four? And so on: whenever you make a choice, you face a question. In this talk, I outline a systematic account of the role that questions play in decision-making, in the form of a new, inquisitive decision theory.

Inquisitive decision theory can account for many ordinary patterns of behaviour that classical decision theory does not capture. In particular, we can account for the distinction between recognition and recall, and for belief states that are not closed under deduction. The theory builds on a converging set of insights about the role of questions from epistemology and the philosophy of language, semantics, pragmatics, psychology, decision theory and the metaphysics of propositions.


[Note: This handout is for the benefit of those who want to keep track of all the formal details. In the presentation I will focus on the conceptual and intuitive ideas behind these definitions.]

## I. Individual Beliefs

## Classical View of Belief and Action (informal statement)

1) Classical Belief. A belief is the possession of a piece of information about the world.
2) Classical Decisions. And manifests itself in behaviour as a general disposition to act on that information.

## Inquisitive View of Belief and Action (informal statement)

A) Inquisitive Belief. A belief is the possession of an answer to a particular question.
B) Inquisitive Decisions. And manifests itself in behaviour as a disposition to act on that answer when faced with that question.

## Formal definitions

" $\Omega$ " denotes logical space, the set of all possible worlds. Throughout we assume $\Omega$ is finite.

- An (intensional) proposition is a a subset of $\Omega$.
- Aquestion is a partition of $\Omega$.
- Any set of partition cells of a question is an answer to that question.
- A question-dependent proposition or quizposition is an ordered pair $\langle\mathrm{Q}, \mathrm{A}\rangle$ of a question Q and an answer A to Q .
- An option is a function $\mathbf{a}: \Omega \rightarrow \mathbb{R}$ from possible worlds to utility values. (Concretely, $\mathbf{a}(w)$ represents the utility that, at $w$, would have been obtained if the option a had been chosen.)
- A decision situation or choice is a finite set of options.
- The choice $\mathbf{C}$ raises the question Q , or Q addresses $\mathbf{C}$, just in case for every option $\mathbf{a} \in \mathbf{C}$, and any complete answer $q \in \mathrm{Q}$, the outcome $\mathbf{a}(w)$ takes on a constant value for all $w \in \mathrm{q}$, denoted ' $\mathbf{a}(\mathrm{q})^{\prime}$. An agent faces the question Q whenever they make a choice that raises Q .
- The option a (strictly) $p$-dominates the option $\mathbf{b}$ just in case $\mathbf{a}(w)>\mathbf{b}(w)$ for all $w \in p$.
- Suppose $\mathbf{a}$ and $\mathbf{b}$ are options in a decision situation that raises $Q$, and $A$ is an answer to $Q$. Then option a (strictly) A-dominates option $\mathbf{b}$ just in case $\mathbf{a}(q)>\mathbf{b}(q)$ for all $q \in A$.


## Classical View of Belief and Action (rough formal gloss)

3) Classical Belief. Belief is a relation between agents and intensional propositions
4) Classical Decisions. An agent believing the proposition $p$ is always disposed to choose the p-dominant option, if there is one.

## Inquisitive View of Belief and Action (rough formal gloss)

C) Inquisitive Belief. Belief is a relation between agents and quizpositions
D) Inquisitive Decisions. An agent believing the quizposition $\langle\mathrm{Q}, \mathrm{A}\rangle$ is disposed to choose the A-dominant option, if there is one, in any decision situation that raises Q .

## II. Belief States

Classical Doxastic Coherence, "The Map by Which We Steer" (Ramsey 1926)
5) An agent's beliefs form a classical information state; that is, they satisfy these two closure conditions:
i) Closure under entailment. If an agent believes $p$, and $p$ entails $q$, then they believe $q$.
ii) Closure under conjunction. If an agent believes $p$ and also $q$, then they believe $p \cap q$.
6) An agent's beliefs are consistent: there is a possible world at which they are all true.

## Quizpositional parts

- The conjunction of two questions $Q$ and $Q^{\prime}$ is the question $Q Q^{\prime}:=\left\{q \cap q^{\prime}: q \in Q, q^{\prime} \in Q^{\prime}\right\} \backslash\{\varnothing\}$. The conjunction of a $Q$-answer $A$ and a $Q^{\prime}$-answer $A^{\prime}$ is $A A^{\prime}:=\left\{a \cap a^{\prime}: a \in A, a^{\prime} \in A^{\prime}\right\} \backslash\{\varnothing\}$, an answer to $\mathrm{QQ}^{\prime}$. The conjunction of $\langle\mathrm{Q}, \mathrm{A}\rangle$ and $\left\langle\mathrm{Q}^{\prime}, \mathrm{A}^{\prime}\right\rangle$ is the quizposition $\left\langle\mathrm{QQ}^{\prime}, \mathrm{AA}^{\prime}\right\rangle$.
- A question $Q$ contains a question $Q^{\prime}$ if and only if every complete $Q^{\prime}$-answer $q \in Q^{\prime}$ is equal to a union of complete Q -answers (this is true just in case $\mathrm{QQ}^{\prime}=\mathrm{Q}$ ). We can also say that $\mathrm{Q}^{\prime}$ is part of Q , or that Q is at least as fine-grained as $\mathrm{Q}^{\prime}$, or that Q entails $\mathrm{Q}^{\prime}$.
- A quizposition $\langle Q, A\rangle$ (classically) entails $\left\langle Q^{\prime}, A^{\prime}\right\rangle$, if and only if $\cup A \subseteq \cup A^{\prime}$.
- A quizposition $\langle\mathrm{Q}, \mathrm{A}\rangle$ contains $\left\langle\mathrm{Q}^{\prime}, \mathrm{A}^{\prime}\right\rangle$, if and only if Q contains $\mathrm{Q}^{\prime}$ and $\langle\mathrm{Q}, \mathrm{A}\rangle$ classically entails $\left\langle Q^{\prime}, A^{\prime}\right\rangle$ (this is true just in case $\left\langle Q Q^{\prime}, A A^{\prime}\right\rangle=\langle Q, A\rangle$ ). We can also say that $\left\langle Q^{\prime}, A^{\prime}\right\rangle$ is part of $\langle Q, A\rangle$.



## Inquisitive Doxastic Coherence, "The Web of Questions"

E) An agent's beliefs form an inquisitive information state; that is, they satisfy these two closure conditions:
i) Closure under parthood. If an agent believes $\langle\mathrm{Q}, \mathrm{A}\rangle$, and $\langle\mathrm{Q}, \mathrm{A}\rangle$ contains $\left\langle\mathrm{Q}^{\prime}, \mathrm{A}^{\prime}\right\rangle$ as a part, then they believe $\left\langle\mathrm{Q}^{\prime}, \mathrm{A}^{\prime}\right\rangle$.
ii) Limited closure under conjunction. If an agent believes $\langle Q, A\rangle$ and also $\left\langle Q^{\prime}, A^{\prime}\right\rangle$, and $Q^{\prime}$ is part of Q , then they believe the conjunction $\left\langle\mathrm{Q}, \mathrm{AA}^{\prime}\right\rangle$.
F) An agent's beliefs are coherent: that is, they do not include contradictions $\langle\mathrm{Q}, \varnothing\rangle$. Coherence in this sense does not entail consistency.


DIAGRAM 2: MUTUALLY INCONSISTENT BUT COHERENT VIEWS


## III. Credences

Recall that for simplicity, the set of possible worlds $\Omega$ is assumed to be finite.

## Probabilities and Expected Values

- A classical probability is a function $\operatorname{Pr}: \mathcal{P}(\Omega) \rightarrow[0,1]$ from propositions to the unit interval, subject to the following two conditions:
- Normalisation: $\operatorname{Pr}(\Omega)=1$
- Additivity: For any disjoint propositions $p, q, \operatorname{Pr}(p \cup q)=\operatorname{Pr}(p)+\operatorname{Pr}(q)$
- Let $\operatorname{Pr}$ be a classical probability and a any option. Then a's classical expected value given $\operatorname{Pr}$ is

$$
\mathcal{E}_{\operatorname{Pr}}(\mathbf{a}):=\Sigma_{w \in \Omega} \operatorname{Pr}(\{w\}) \cdot \mathbf{a}(w)
$$

- An inquisitive probability is a partial function $\operatorname{Pr}: \mathcal{Q}(\Omega) \times \mathcal{P}(\Omega) \rightarrow[0,1]$ from quizpositions to the unit interval, subject to the following conditions:
- Inquisitive Domain: There is a non-empty set of questions $\mathrm{D}_{\mathrm{Pr}}$ such that $\operatorname{Pr}(\mathrm{Q}, \mathrm{A})$ is defined if and only if $\mathrm{Q} \in \mathbf{D}_{\mathrm{Pr}}$ and A is an answer to Q . $\mathbf{D}_{\mathrm{Pr}}$ is closed under entailment.
- Normalisation: For all $\mathrm{Q} \in \mathrm{D}_{\mathrm{Pr}}, \operatorname{Pr}(\mathrm{Q}, \mathrm{Q})=1$
- Additivity: For all $\mathrm{Q} \in \mathrm{D}_{\text {Pr }}$ and disjoint $\mathrm{A}, \mathrm{B} \subseteq \mathrm{Q} \operatorname{Pr}(\mathrm{Q}, \mathrm{A} \cup \mathrm{B})=\operatorname{Pr}(\mathrm{Q}, \mathrm{A})+\boldsymbol{\operatorname { P r }}(\mathrm{Q}, \mathrm{B})$
- Coherence: If $\langle Q, A\rangle$ and $\left\langle Q^{\prime}, A^{\prime}\right\rangle$ are intensionally equivalent quizpositions $\left(U A=U A^{\prime}\right)$ and $\mathrm{Q}, \mathrm{Q}^{\prime} \in \mathrm{D}_{\mathrm{Pr}}$, then $\operatorname{Pr}(\mathrm{Q}, \mathrm{A})=\operatorname{Pr}\left(\mathrm{Q}^{\prime}, \mathrm{A}^{\prime}\right)$.
- Let $\operatorname{Pr}$ be an inquisitive probability, let $\mathbf{C}$ be a choice raising the question $\mathrm{Q} \in \mathrm{D}_{\text {Pr }}$, and let a be an option in C. Then a's inquisitive expected value given $\operatorname{Pr}$ is

$$
\mathcal{E}_{\operatorname{Pr}( }(\mathbf{a}):=\Sigma_{q \in \mathrm{Q}} \operatorname{Pr}(\mathrm{Q},\{\mathrm{q}\}) \cdot \mathbf{a}(\mathrm{q})
$$

## Representation theorems

- An agent discerns dominant options just in case they are disposed to avoid strictly dominated options in any decision situation.
- An agent discerns better outcomes just in case they are disposed to pick the best outcome in any choice between constant options. (Where a constant option is an option that assigns the same utility to every possible world.)
- An agent maximises utility with respect to the inquisitive probability Pr just in case (i) in every decision situation that raises a question $\mathrm{Q} \in \mathrm{D}_{\mathrm{Pr}}$ the agent performs the option a that maximises the value of $\mathcal{E}_{\mathrm{Pr}}(\mathbf{a})$ and (ii) no extension of $\operatorname{Pr}$ to a larger domain has property (i).

Classical Representation Theorem. Any agent who discerns dominant options is disposed to maximise classical expected utility with respect to some unique classical probability.

Inquisitive Representation Theorem. Any agent who discerns better outcomes is disposed to maximise inquisitive expected utility with respect to some unique inquisitive probability.

General Representation Theorem. Any agent has well-defined inquisitive credences about all and only those questions Q such that the agent is disposed to avoid dominated options when faced with Q .

