Questions in *Action*

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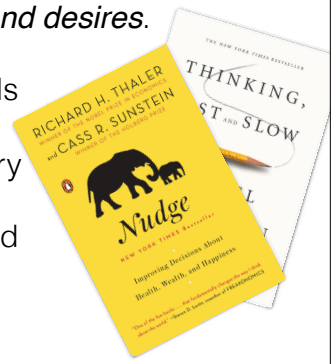
Background & Motivation

- In economics and cognitive science, decision theory is used to ***predict and explain the behaviour of agents in terms of their beliefs and desires***.
- There are big debates in both fields about the rationality assumptions underlying classical decision theory
- The classical theory is too idealised to yield reliable predictions.



Part I: Individual Beliefs

Part II: Belief States

Part III: Credences

Classical View of Belief and Action

Classical Belief. **A belief is the possession of a piece of information about the world.**

Classical Decisions. **And manifests itself as a general disposition to act on that information.**

5

Inquisitive belief

Classical Belief. A belief is the possession of a piece of information about the world.

Inquisitive Belief. **A belief is the possession of an answer to a particular question.**

(Dretske 1981; Schiffer 1992; Aloni 2001; Schaffer 2009; Blaauw 2013; Pérez Carballo 2016; Hawthorne, Rothschild and Spectre 2016; Simons, Beaver, Roberts and Tonhauser 2017; Yalcin 2018; Holguín f.c.)

6

Inquisitive belief

Rather than **intensional propositions** or sets of possible worlds, the contents of belief are **question-dependent propositions** or **quizpositions**, where:

- A **question** Q is a partition of logical space Ω
- Any subset A of Q is an **answer** to Q
- A **quizposition** is an ordered pair $\langle Q, A \rangle$ of a question Q and an answer A to that question

7

Inquisitive Decisions

Observation:

Choices confront agents with questions

- Ordinary discourse about difficult decisions makes frequent reference to questions ("Facing questions" / "Asking yourself questions")
- In some languages, the cognates of "question" mean "dilemma": Cuestión / Questione / Kwestie

8

Inquisitive Decisions

Classical Decisions. A belief that *p* manifests itself as a **general** disposition to act on the information that *p*.

Inquisitive Decisions. **A belief that *A* in answer to *Q* manifests itself as a disposition to act on the answer *A* when making any decision that raises the question *Q*.**

Facing Questions

	Chicken	Beef	Herring
White	1	−1	1
Red	0	1	−1
Rosé	.5	0	−1

Facing Questions

Definition. An **option** is a function **a** from possible worlds to utility values. Concretely, **a**(*w*) represents the utility that, at *w*, would have been obtained had the option **a** been chosen.

Definition. A **decision situation** or **choice** is a finite set of options.

	Chicken	Beef	Herring
White	1	−1	1
Red	0	1	−1
Rosé	.5	0	−1

Facing Questions

Definition. The choice **C** **raises** the question *Q*, or *Q* **addresses** **C**, just in case every option **a** in **C** takes on a single value within each cell *q* of *Q*, written '**a**(*q*)'.

	Chicken	Beef	Herring
White	1	−1	1
Red	0	1	−1
Rosé	.5	0	−1

Facing Questions

	The word /drɛmt/ ends on -MT	*The word /drɛmt/ ends on -MED*	*The word /drɛmt/ ends on -MNT*	…
Write "MT"	1	0	0	…
Write "MED"	0	1	0	…
…	…	…	…	…

Facing Questions

	Only /drɛmt/ ends on -MT	*Only /ʌn'kɛmt/ ends on -MT*	*Only /prɒmt/ and /drɛmt/ end on -MT*	…
Answer /drɛmt/	25¢	0	25¢	…
Answer /ʌn'kɛmt/	0	25¢	0	…
Answer /prɒmt/	0	0	25¢	…
…	…	…	…	…

Facing Questions

Classical Decisions

Definition. The option **a** **(strictly)** *p*-**dominates** the option **b** just in case **a**(*w*) > **b**(*w*) for all *p*-worlds *w*.

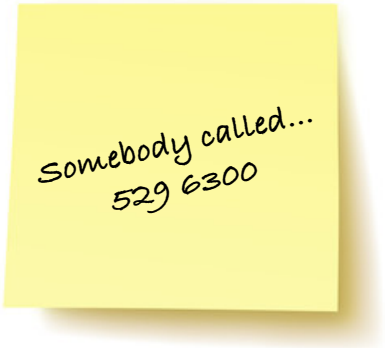
Classical View. If an agent believes the proposition *p*, then whenever the agent makes a choice, they will choose the *p*-dominant option if there is one.

Inquisitive Decisions

Definition. The option **a** **(strictly)** *A*-**dominates** the option **b** just in case **a**(q) > **b**(q) for all cells q in A.

If an agent believes the quizposition $\langle Q, A \rangle$, then when the agent makes a choice **C** ***raising the question Q***, they will choose choose an *A*-dominant option in **C** if there is one.

Recognition vs. Recall



To whom does the phone number 529 6300 belong?

Recognition vs. Recall

What is Romeo's phone number?

Part I: Individual Beliefs

Part II: Belief States

Part III: Credences

Classical Doxastic Coherence

- Our individual beliefs are just aspects of a single global world view.
- "The map by which we steer" (Ramsey 1926)
- Since our beliefs fit together into this one global picture, they must all cohere with each other.

Classical Doxastic Coherence

A classical agent's beliefs satisfy these closure conditions:

a) *Closure under entailment*

b) *Closure under conjunction*

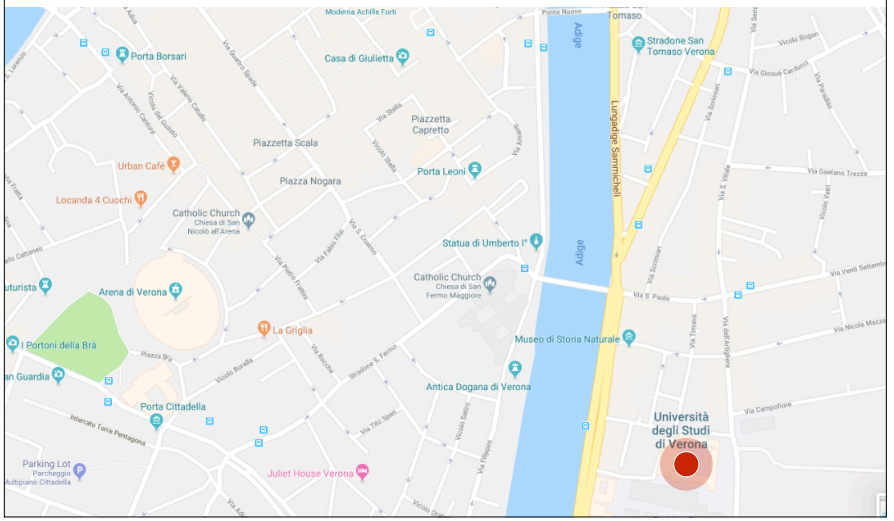
A **classical information state** is a collection of intensional propositions satisfying these two conditions.

Classical Doxastic Coherence

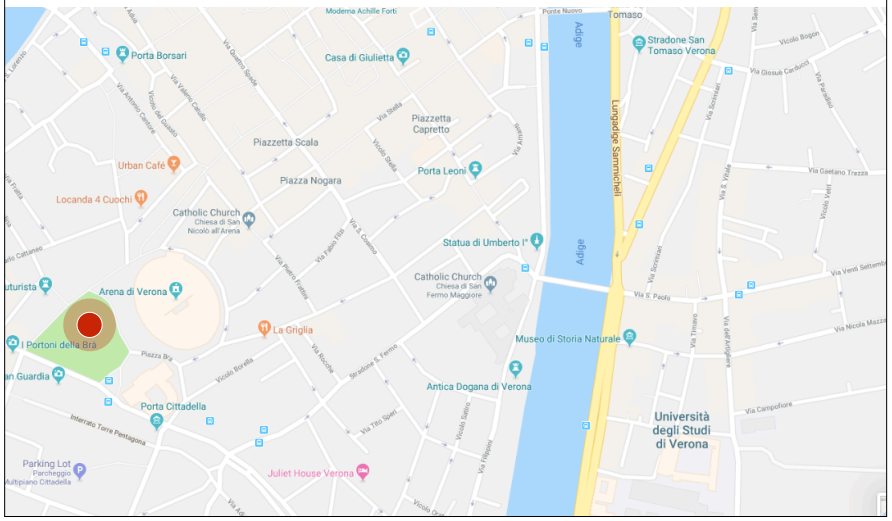
Besides being a classical information state, the belief state must be **consistent**: that is, there is some possible world *w* such that all of the agent's beliefs are true relative to *w*.

(If the agent's beliefs were not consistent, then given closure under conjunction, the agent would believe a contradiction. And given what we said above, that would yield conflicting predictions about the agent's behaviour).

Classical Doxastic Coherence



Classical Doxastic Coherence



Quizposition Mereology

Definition. A question *Q* **contains** *Q′* if and only if *Q* makes at least as many distinctions between possible worlds as *Q′*.

We can also say that *Q′* is **part of** *Q*, or that *Q* is ***at least as fine-grained*** as *Q′*, or that *Q* ***entails*** *Q′*.

Quizposition Mereology

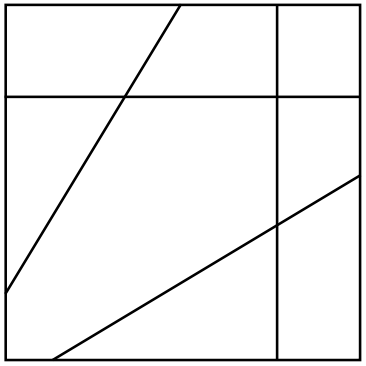
Definition. A quizposition ⟨*Q*, *A*⟩ **contains** ⟨*Q′*, *A′*⟩, or ⟨*Q′*, *A′*⟩ is **part of** ⟨*Q*, *A*⟩, if and only if

i. ⟨*Q*, *A*⟩ entails ⟨*Q′*, *A′*⟩ ***and***

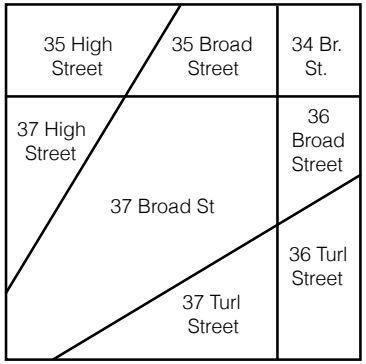
ii. *Q* contains *Q′*.

(A quizposition ⟨*Q*, *A*⟩ ***entails*** ⟨*Q′*, *A′*⟩ if and only if at every world where ⟨*Q*, *A*⟩ is true, ⟨*Q′*, *A′*⟩ is true as well.)

Quizpositional Mereology



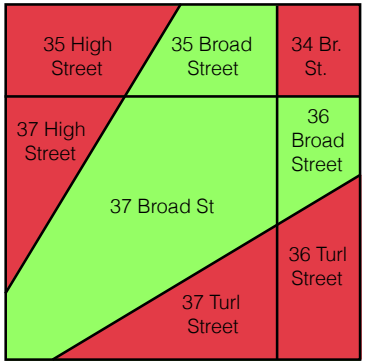
Quizpositional Mereology



What is Jenny's address?

29

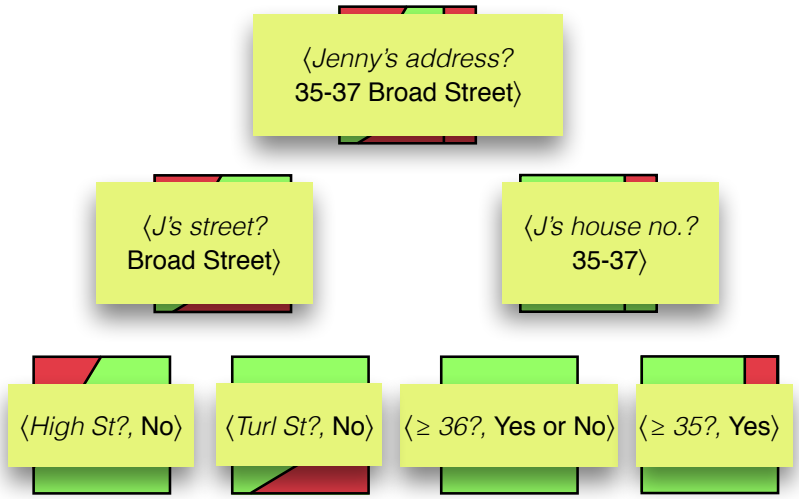
Quizpositional Mereology



⟨*What is Jenny's address?*, Between 35 and 37 Broad Street⟩

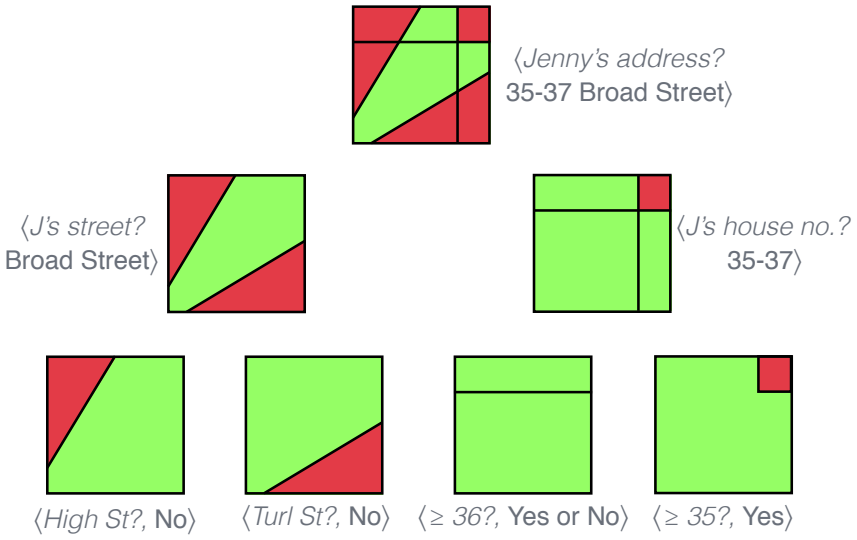
30

Quizpositional Mereology



31

Quizpositional Mereology



32

Parts and Choices

If some question *Q* addresses a choice, then any bigger question that contains *Q* as a part addresses that choice as well

	Chicken	Beef	Herring
White	1	–1	1
Red	0	1	–1
Rosé	0.5	0	–1

Parts and Choices

If some question *Q* addresses a choice, then any bigger question that contains *Q* as a part addresses that choice as well

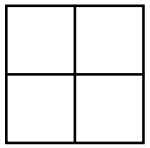
	Chicken	Beef	Pacific Herring	Atlantic Herring
White	1	–1	1	1
Red	0	1	–1	–1
Rosé	0.5	0	–1	–1

Parts and Choices

If some question *Q* addresses a choice, then any bigger question that contains *Q* as a part addresses that choice as well

	Chicken & salad	Chicken, no salad	Beef	Pacific Herring	Atlantic Herring
White	1	1	–1	1	1
Red	0	0	1	–1	–1
Rosé	0.5	0.5	0	–1	–1

Parts and Choices



Parts and Choices

⟨*What is the biggest and the second biggest city in Brazil?*
Rio and São Paolo are the two biggest cities⟩

Rio	Brasí-lia
São Paolo	Salva-dor

⟨*What is the biggest city in Brazil?*
… ⟩

Parts and Choices

⟨*What is the biggest and the second biggest city in Brazil?*
Rio and Sao Paolo are the two biggest cities⟩

Rio	Brasí-lia
São	va-dor

⟨*What is the biggest city in Brazil?*
It's Brasília ⟩

Parts and Choices

⟨*What is the biggest and the second biggest city in Brazil?*
Rio and São Paolo are the two biggest cities⟩

Rio	Brasí-lia
São Paolo	Sa…

⟨*What is the biggest city in Brazil?*
It's not Salvador ⟩

Parts and Choices

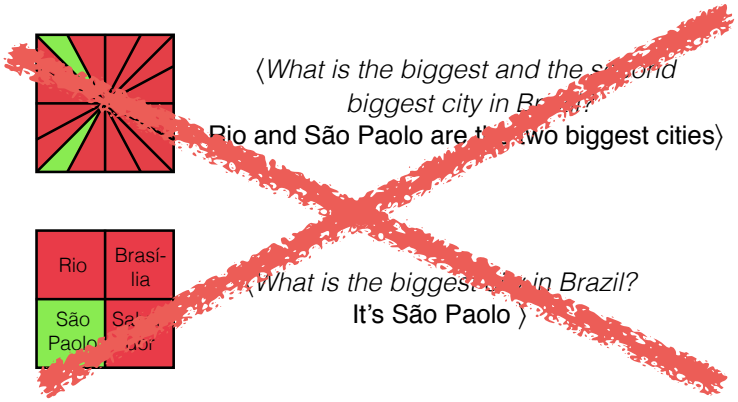
⟨*What is the biggest and the second biggest city in Brazil?*
Rio and São Paolo are the two biggest cities⟩

Rio	Brasí-lia
São Paolo	Salva-dor

⟨*What is the biggest city in Brazil?*
≥ [It's Rio or Sao Paolo] ⟩

Beliefs are closed under *parthood*

Parts and Choices



⟨*What is the biggest and the second biggest city in Brazil?*
Rio and São Paolo are the two biggest cities⟩

⟨*What is the biggest city in Brazil?*
It's São Paolo ⟩

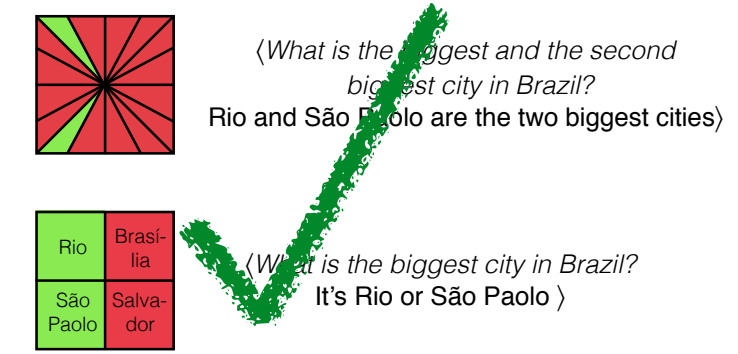
Parts and Choices

⟨*What is the biggest and the second biggest city in Brazil?*
≥ [São Paolo is the biggest city] ⟩

⟨*What is the biggest city in Brazil?*
It's São Paolo ⟩

Beliefs are closed under some conjunctions

Parts and Choices



⟨*What is the biggest and the second biggest city in Brazil?*
Rio and São Paolo are the two biggest cities⟩

⟨*What is the biggest city in Brazil?*
It's Rio or São Paolo ⟩

Inquisitive Doxastic Coherence

A classical agent's beliefs satisfy these closure conditions:

a) *Closure under **parthood**.*

b) ***Limited** closure under conjunction.*

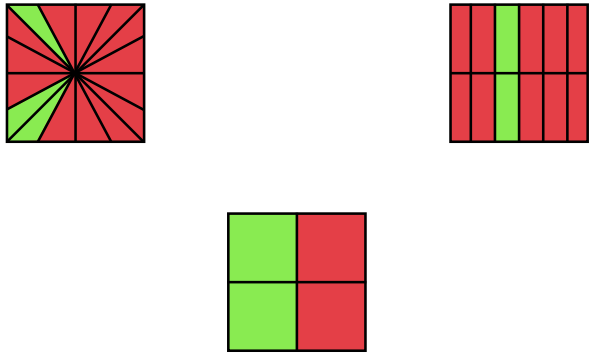
An **inquisitive information state** is a collection of quizpositions satisfying these conditions.

Inquisitive Doxastic Coherence

Besides being a classical information state, the belief state must be **coherent**: they cannot include contradictions $\langle Q, \varnothing \rangle$.

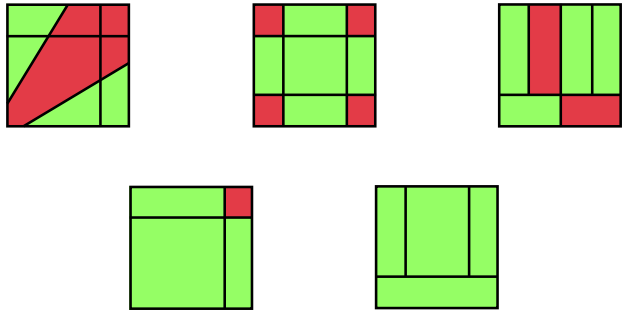
Coherence does not imply consistency.

The Web of Questions

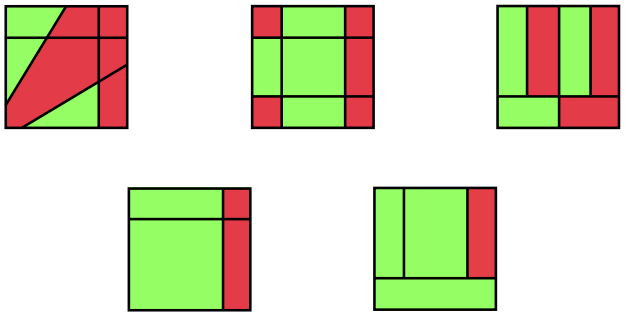


The Web of Questions

The Web of Questions



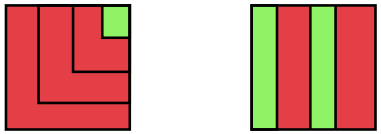
The Web of Questions



49

The Web of Questions

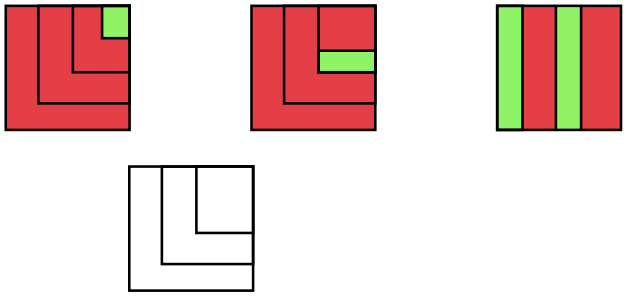
We allow for the occurrence of **inconsistency**:



50

The Web of Questions

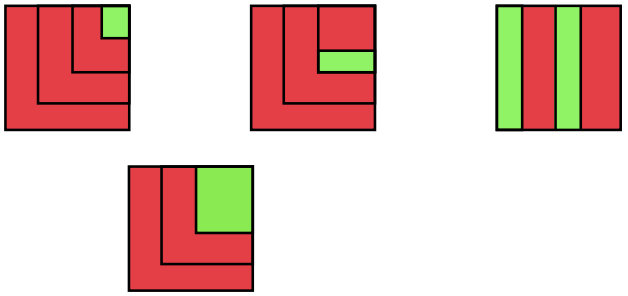
Even between overlapping questions



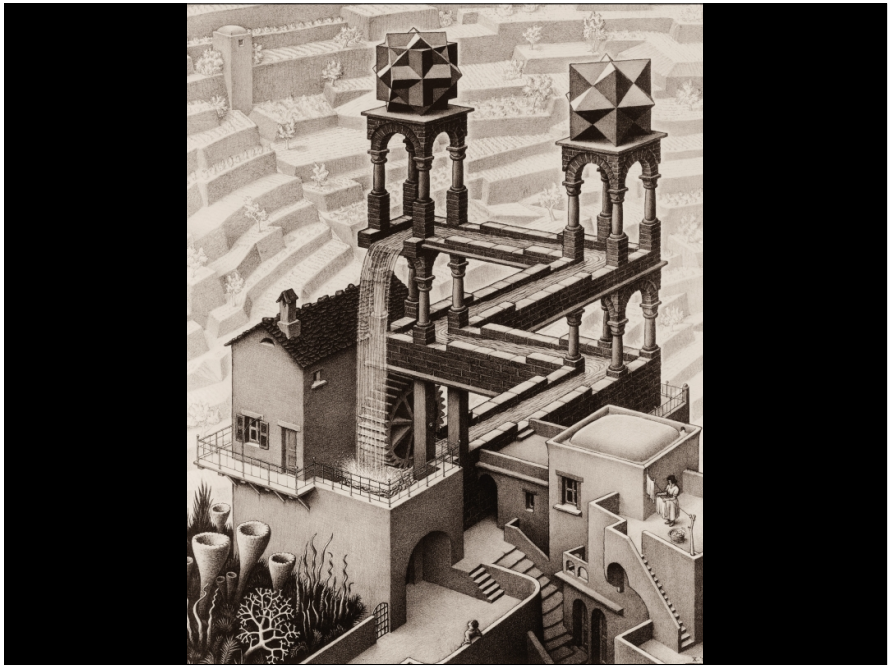
51

The Web of Questions

Even between overlapping questions



52



Part I: Individual Beliefs

Part II: Belief States

Part III: Credences

Classical Decision Theory

Definition. A **classical probability** is a function $\mathsf{Pr}: \mathcal{P}(\Omega) \to [0, 1]$ from propositions to the unit interval, such that:

- *Normalisation.* $\mathsf{Pr}(\top) = 1$
- *Additivity.* If p and q are disjoint $\mathsf{Pr}(p \cup q) = \mathsf{Pr}(p) + \mathsf{Pr}(q)$

Definition. Let Pr be a classical probability. Then the **classical expected utility** of any option a given Pr is:

$$\mathcal{E}_{\mathsf{Pr}}(\mathsf{a}) \quad =_{\mathrm{df}} \quad \Sigma_{w \in \Omega} \ \mathsf{Pr}(\{w\}) \cdot \mathsf{a}(w)$$

Classical Decision Theory

Definition. An agent **discerns dominant options** just in case they are disposed never to choose a strictly dominated option in any decision situation.

Classical Decision Theory

Classical Representation Theorem. An agent who discerns dominant options maximises expected utility with respect to some unique classical probability **Cr**.

Inquisitive Decision Theory

Definition. An **inquisitive probability** is a partial function $\mathsf{Pr}\colon \mathcal{Q}(\Omega) \times \mathcal{P}(\Omega) \rightarrow [0, 1]$ from quizpositions to the unit interval, such that:

- *Inquisitive Domain*. There is a set of questions $\mathbf{D}_{\mathsf{Pr}}$, called the **domain** of **Pr**, such that $\mathsf{Pr}(Q, A)$ is defined if and only if $Q \in \mathbf{D}_{\mathsf{Pr}}$ and A is an answer to Q. $\mathbf{D}_{\mathsf{Pr}}$ is closed under entailment.
- *Coherence*. For any Q, $Q' \in \mathbf{D}_{\mathsf{Pr}}$ that share a common part R, and any answer A to R, $\mathsf{Pr}(Q, A) = \mathsf{Pr}(Q', A)$.
- *Normalisation and Additivity*. For any $Q \in \mathbf{D}_{\mathsf{Pr}}$, $\mathsf{Pr}(Q, \top) = 1$, and if A and B are disjoint, $\mathsf{Pr}(Q, A \cup B) = \mathsf{Pr}(Q, A) + \mathsf{Pr}(Q, B)$

Inquisitive Decision Theory

Definition. Let **Pr** be a classical probability with $Q \in \mathbf{D}_{\mathsf{Pr}}$. Then the **inquisitive expected utility** of any option **a** in a decision problem that raises Q is:

$$\mathcal{E}_{\mathsf{Pr}}(\mathsf{a}) \quad =_{\mathrm{df}} \quad \Sigma_{q \in Q}\ \mathsf{Pr}(\{q\}) \cdot \mathsf{a}(q)$$

Inquisitive Decision Theory

Definition. An agent **discerns better outcomes** just in case they are disposed to pick the best outcome in any choice between constant options.

(A **constant option** is an option that assigns the same utility value to every possible world.)

Inquisitive Decision Theory

Inquisitive Representation Theorem. An agent who discerns better outcomes maximises expected utility with respect to some unique inquisitive probability **Cr**.

Inquisitive Decision Theory

General Representation Theorem. Any agent has well-defined inquisitive credences about all and only those questions *Q* such that the agent is disposed never to pick a dominated option when faced with *Q*.

Inquisitive Decision Theory

General Representation Theorem. Any agent has well-defined inquisitive credences about all and only those questions *Q* such that the agent is disposed never to pick a dominated option when faced with *Q*.

Classical Representation Theorem. An agent who discerns dominant options maximises expected utility with respect to some unique classical probability **Cr**.

Inquisitive Representation Theorem. An agent who discerns better outcomes maximises expected utility with respect to some unique inquisitive probability **Cr**.

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