Problem Set 2: Problems of Induction

Always answer in complete sentences. Due by February 28th at 8pm.

1.A. Define each of the following terms as accurately as you can. Use your own words, and give examples where helpful.

   a. **Conditional Probability**
   b. **Confirmation**
   c. **Chance**
   d. **Likelihood**
   e. **Entailment**
   f. **The Kolmogorov Axioms** (list them!)
   g. **The Principal Principle**
   h. **Credence**
   i. **Priors**
   j. **The Problem of Induction**

B. Explain what each of the following four terms mean, and how they relate to one another: **Bayes' Theorem**, **Bayes' Rule**, the **Bayesian Multiplier** and **Bayesianism** (consult Michael Strevens' notes if you need help).

2. This is a question about ***probabilisitic independence***.

   a. Write down the mathematical definition of probabilistic independence.

   b. Three six-sided dice A, B, and C are about to be thrown. Which of the following pairs of events have independent ***chances*** of being realised? Explain, in each case, why these chances are or aren't independent. Do **not** assume the dice are fair.

      i. *Die A and C land on a 5* and *Die B lands on a 5*
      ii. *Die A beats die B* and *Die B beats die C*
      iii. *Die B beats die A* and *Die A lands on an even number*

   c. Assuming the dice *are* fair, check that your answers in (b) conform to the mathematical definition of independence, by calculating the probability of each of the six individual events and each of the three conjunctions of events.

   (*For example, in the case of (i), you should find the probability that 'Die A and C land on a 5', the probability that 'Die B lands on a 5', and the probability that 'Die A, C and B land on a 5', and then determine whether the two former probabilities are indeed independent of one another*)

d. A coin is about to be tossed ten times. Which of these pairs of events should you have independent ***credences*** about? Explain your answers. Do **not** assume the coin is fair.

i.	*Five out of ten tosses land Heads*	and	*The first toss lands Heads*
ii.	*The first six tosses land Heads*	and	*The seventh toss lands Heads*
iii.	*Three Heads in the first six tosses*	and	*Two Heads in the final four*

e. Give a non-mathematical characterisation of what makes two events probabilistically independent. Would there be a difference in your characterisation of independent *chances* versus independent *credences*? Explain.

3. To do this question, begin by looking up equation (2) on p. 60 the Dawid and Gillies paper about Hume on miracles.

   a. Explain what all the different letters in the equation stand for.

   b. Show how this equation can be derived from the axioms of probability and the definition of a conditional probability $\Pr(X \mid Y) =_{\text{df}} \Pr(XY)/\Pr(Y)$.

   c. Here's a story:

   > Your friend Octavia says to you, "I just flipped a fair coin and on the first 20 tosses got the following sequence: HTHHTHHHTTTHTTTTHHTH" (a normal, unpatterned sequence). You reply "Cool". Then Xerxes (another friend) says "I just flipped a fair coin and on the first 10 tosses got the following sequence: HHHHHHHHHH". You reply "Baloney!"
   >
   > Xerxes is offended. He huffs "Octavia's sequence had probability $1/2^{20}$ and you believed her. But my sequence had probability $1/2^{10}$. So my sequence was thousands of times more likely than hers. Yet you believed her and you did not believe me. You are a horrible friend!"

   In prose, give the clearest explanation you can of why it was reasonable for you to believe Octavia but not Xerxes. Be sure to mention how your explanation deals with Xerxes's claim that his reported sequence was more likely.

   d. Use equation (2) to back up your answer to (c)

4. Imagine you visit the little-known town of Vlastivosk for the first time. You don't know anything about Vlastivosk, except that it is nearby two other major towns. Upon your arrival at the airport, you see three bus schedules. But the names of the towns are blurred out, so you don't know which of these schedules is for Vlastivosk. The first schedule lists ten bus lines numbered 1-10, the second lists thirty lines and the final schedule lists sixty. This gives you excellent reason to believe that one of the following three hypotheses is true:

   h_{10}. There are ten bus lines in Vlastivosk, numbered 1, 2, 3 … 10.

   h_{30}. There are thirty bus lines in Vlastivosk, numbered 1, 2, 3 … 30.

   h_{60}. There are fifty bus lines in Vlastivosk, numbered 1, 2, 3 … 60.

   Assume that you divide your credences equally between these three hypotheses — that is to say, you give each one a prior credence of ⅓.

   a. Walking through a random street in Vlastivosk, you see a bus coming towards you. It says "9" on the front. Explain in prose why this should affect your credences about h_{10}, h_{30} and h_{60}. Which hypotheses do you think will be confirmed by the observation, and which ones do you think will be disconfirmed?

   b. Assuming Bayes' Rule, calculate what your credences about h_{10}, h_{30} and h_{60} ought to be after making this observation.
   (*To denote the proposition that the first bus you spotted was a 9, write "e* [1, 9]*". In general, use the notation $e\,[m, n]$ for the proposition that the m^{th} bus you spotted is bus n.*)

   c. *Bonus (extra credit)*. Explain why the priors about h_{10}, h_{30} and h_{60} make a difference to whether h_{30} is confirmed or disconfirmed by the observation in (a). Give one alternative set of priors on which $e\,[1, 9]$ counts as confirming h_{30}.

   d. Continuing on your stroll through Vlastivosk, the next bus you happen upon is bus 8; after that you see bus 2, bus 9 again and then bus 5. What are your credences about h_{10}, h_{30} and h_{60} after having made these five observations? Assume that each bus sighting corresponds to a fair random draw from the set of Vlastivoskan buslines.
   (*So in our notation, the new pieces of evidence acquired are* $e\,[2, 8]$, $e\,[3, 2]$, $e\,[4, 9]$ and $e\,[2, 5]$.)

   e. The next five buses are 1, 9, 17, 25 and 7. Still assuming that the sightings are random draws, calculate your posterior credences in h_{10}, h_{30} and h_{60} after all ten observations.

   f. In (a-d), I told you to assume that the bus sightings were random. Explain why these observations seem to indicate that your sampling may in fact have been biased or unfair. What could explain the bias?

   g. If you do take the possibility of bias into account, how would that affect your credences in h_{10}, h_{30} and h_{60}? (You do not need to do any calculations here.)

5. David Hume argued that predictions about the future on the basis of past observations can never be fully justified on the basis of Reason alone, because all inductive reasoning relies on an unjustified hypothesis that the future will resemble the past. Hume called this the hypothesis of the *Uniformity of Nature*, and he argued we had no choice but to accept it uncritically, on faith. (By a justification based on Reason alone, Hume means a justification independent of experience, the way mathematical theorems are justified independently of experience.)

   a. Explain why Hume thought it was impossible to establish the hypothesis of the Uniformity of Nature deductively, and why he thought it was impossible to establish the hypothesis inductively. [*A good explanation will take at least four to five sentences.*]

   Some apriorist Bayesians claim that Hume was wrong. Using the example from question (4) as a case in point, they could argue as follows:

   P1. The laws of probability are based on Reason alone.

   P2. From past observations, we just showed, using those laws, that h_{30} is highly probable.

   P3. And this thesis h_{30} in turn entails predictions about the future, for instance that all the buses I will see this week in Vlastivosk will numbered between 1 and 30.

   C. So here is a case where past observations can be shown to support conclusions about the future, without any recourse to the Uniformity of Nature.

   Let's grant that the argument is valid, and examine the premises.

   b. Give one objection to P1 and one reason to believe P1. [*Two sentences will do*.]

   c. P2 is problematic. Besides the laws of probability, list at least two other assumptions that apparently played a role in the derivation in 4. (And as many as you can think of.)

   d. Are any of these assumptions based in part on the hypothesis of the Uniformity of Nature? Do you think they can be justified on the basis of Reason alone? Explain.

6. A standard inductive argument is to extrapolate from the many observed grey elephants to the general conclusion that

   g : All elephants are grey

   Nelson Goodman raised a tenacious problem for all theories of confirmation by pointing out that most rationales that have been proposed for that generalisation to h appear to also justify some incompatible, so-called "gruesome" hypotheses like g^* and g^{**}:

   g^* : Some elephants are grey and some are green. All the elephants first observed on or before the 16th of May, 2038 are grey; and all elephants first observed from that date onwards will be green.

g^{**} : Some elephants are grey and some are purple. All the elephants first observed on or before the 12th of March, 2021 are grey; and all elephants first observed from that date onwards will be purple.

Suppose, for simplicity, that the only relevant evidence about these hypotheses is gathered by finding random elephants and observing their colour. Suppose a Bayesian biologist divides their credences equally between the three hypotheses g, g^* and g^{**}.

a. What prediction will the biologist make about the colour of the first new elephant to be observed in 2040? (What are their credences?)

b. Explain why no number of grey elephants observed in 2020 affects those predictions. (*Hint: Write down the Bayesian multipliers for each hypothesis*.)

c. Explain why the existence of gruesome hypotheses seems to raise a difficulty for Bayesian Confirmation Theory as a model of good scientific reasoning.

d. Recall hypotheses h_{10} and h_{30} and h_{60} from question 4. Write down a gruesome hypothesis h^*_{60} about the buses of Vlastivosk that has the following three properties:

   i) h^*_{60} entails h_{60}

   ii) Any bus sighting today that (dis)confirms h_{30}, (dis)confirms h^*_{60} to the same extent.

   iii) h^*_{60} and h_{30} yield radically different predictions about tomorrow.

e. Is it ever be rational to give positive credence to gruesome hypotheses like g^*, g^{**}, and h^*_{60}? Justify your answer.

f. Do we need to invoke the hypothesis of the Uniformity of Nature in order to protect our ordinary generalisations from the competition of their gruesome rivals? Adduce considerations both ways and make a reasoned judgment. [*At least two or three paragraphs*]