Ramsey on Partial Belief

Dan Hoek — PHI 371 Foundations of Probability and Decision Theory — Princeton — March 2020

What is Probability?

(What is the probability of confirmation theory)

§1: The Frequency Theory

- Probability is Chance

  - This is a good and useful view of probability.

  - But this is not the only useful notion of probability.

  - Ramsey is talking about the notion of probability as it is used in confirmation theory.

§2: Mr. Keynes' Theory

- Probability is **Evidential Probability**

- Probability is a *relation between propositions*: the value $C(H|E)$ represents "the objective degree to which E confirms H"

  - In particular, $C(H|E) = 1$ when E entails H, and $C(H|E) = 0$ when E is inconsistent with H.

- This quantity C obeys the axioms of probability theory

Relation between *C* and Belief

- *C* is an objective quantity, independent of our beliefs

- However, a rational believer should apportion their beliefs to *C*, at least in the following sense:

  - If you are rational and your total evidence is given by *E*, then the degree of confidence you should have in *H* is equal to *C(H/E)*

Ramsey's Objections

- No agreement about what *C* is, even in simple cases.

  - For instance, what is the value of
    C(*The next raven I see is black | This shoe is red*)
    C(*John wears glasses | John has blue eyes*)

- Whenever we have confident judgments about *C*, always seem to go through judgments about confidence, or degrees of belief.

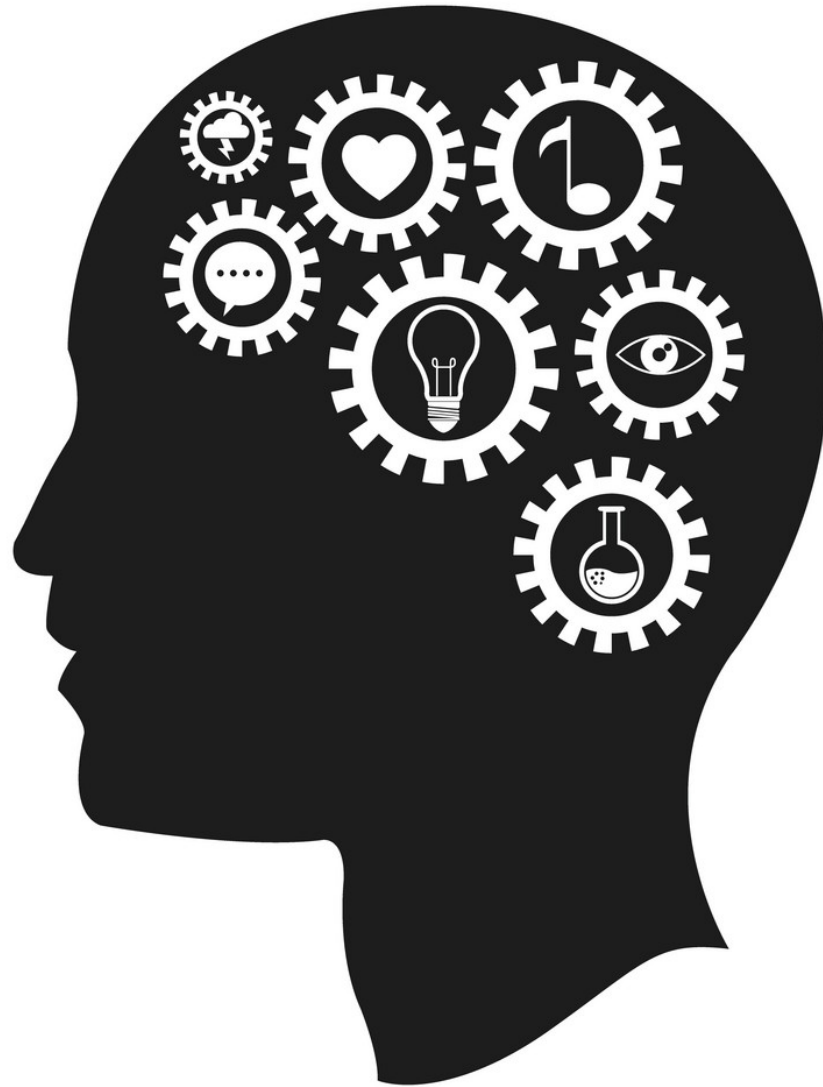# What is belief?

How do we measure beliefs scientifically?

How do we measure the *strength* of a belief?

What is the difference between High and Low Credences?

(Ramsey Terminology)

- "Partial belief" = "Credence"

- "Full belief" = "Credence 1"

- "Jill believes p to degree 2/3" =
 "Jill has credence 2/3 that p"

"Feeling of Belief"

- Perhaps a strong belief is just a belief that we feel more strongly about. This goes back to a Humean idea that the difference between strong and weak beliefs is the *vivacity* with which they appear to the mind.

  - Ramsey notes this would be ***inconvenient*** — it is very difficult to measure a feeling.

  - He also claims it's demonstrably false: "*for the belief which we hold most strongly are often accompanied by practically no feeling at all; **no one feels strongly about things they take for granted**"*

"the nature of the difference between the causes [of belief] is entirely unknown or very vaguely known … *what we want to talk about is the difference between the effects*, which is readily observable and important.

"The difference [between believing more and less firmly] seems to me to lie in *how far we are willing to act on those beliefs*"
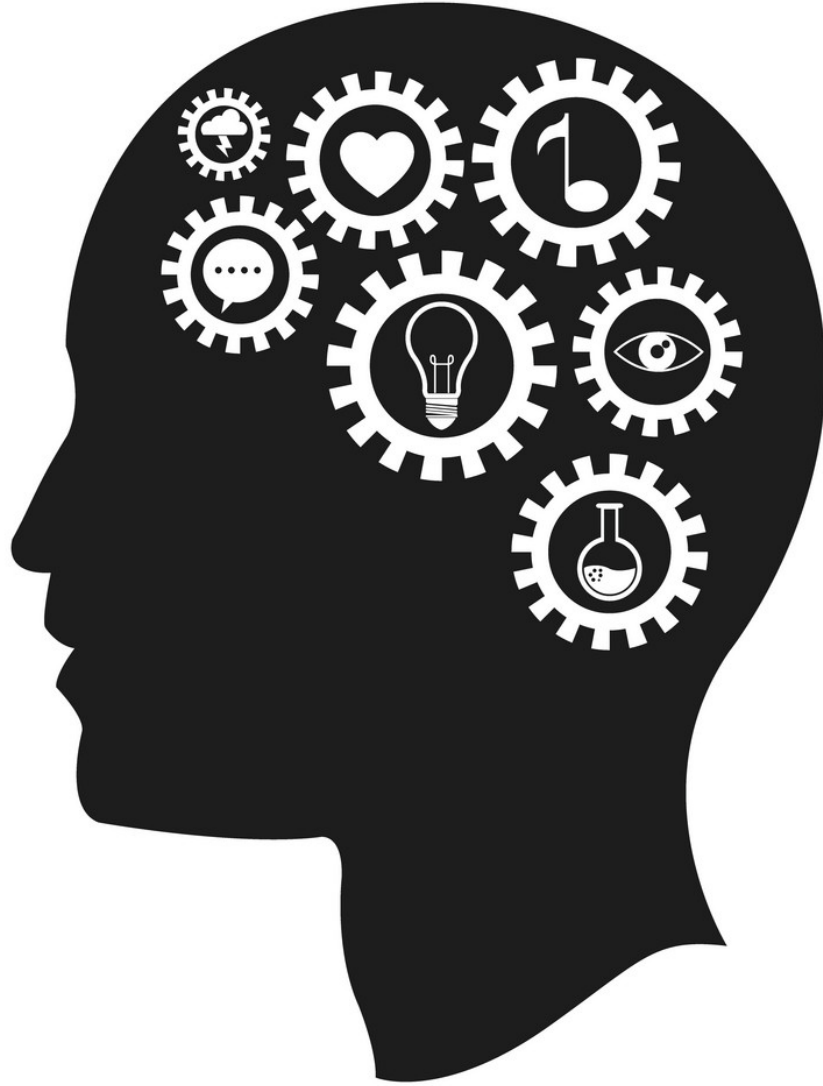
"it is not asserted that a belief is an idea which does actually lead to action, *but one which would lead to action under suitable circumstances*; just as a lump of arsenic is called poisonous not because it actually has killed or will kill anyone, but because it would kill anyone if they ate it."

I. Reducing Credence to Utility

II. Measuring Utility

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"Let us call the things a person ultimately desires 'goods' and let us at first assume that they are numerically measurable and additive."

- "Goods" ≈ "Outcomes"

- "Value"   = "Utility"

Belief-Action Connection

We can put this in a different way. Suppose his degree of belief in p is $\frac{m}{n}$; then his action is such as he would choose it to be if he had to repeat it exactly n times, in m of which p was true, and in the others false. [Here it may be necessary to suppose that in each of the n times he had no memory of the previous ones.]

Belief-Action Connection

Suppose an agent with credence m/n in p makes a choice that depends for its outcome on p. Then the agent will perform the action A that would maximise the utility of the outcome if the agent were to choose A in the same choice situation n times in a row, with p being true in only m cases.

Let us give an instance of the sort of case which might occur. I am at a cross-roads and do not know the way; but I rather think one of the two ways is right. I propose therefore to go that way but keep my eyes open for someone to ask; if now I see someone half a mile away over the fields, whether I turn aside to ask him will depend on the relative inconvenience of going out of my way to cross the fields or of continuing on the wrong road if it is the wrong road. But it will also depend on how confident I am that I am right; and clearly the more confident I am of this the less distance I should be willing to go from the road to check my opinion. I propose therefore to use the distance I would be prepared to go to ask, as a measure of the confidence of my opinion; and what I have said above explaihs how this is to be done. We can set it out as follows: suppose the disadvantage of going x yards to ask is $f(x)$, the advantage of arriving at the right destination is r, that of arriving at the wrong one w. Then if I should just be willing to go a distance d to ask, the degree of my belief that I am on the right road is given by

$$p = 1 - \frac{f(d)}{r - w}.$$

Actions as Bets

all our lives we are in a sense betting. Whenever we go to the station we are betting that a train will really run, and if we had not a sufficient degree of belief in this we should decline the bet and stay at home. The options God gives us are always conditional on our guessing whether a certain proposition is true.

Actions as Bets

- Ramsey describes options in the following general way:

  - Option: Outcome α_1 if p_1 is true, α_2 if p_2 is true, α_3 if p_3... α_n if p_n is true

- Only two types of options are considered in the paper:

  - Unconditional Options: Outcome α whatever happens

  - Binary Bets: Outcome α if p is true, or β if p is false

Actions as Bets

	p_1	p_2	p_3	...	p_n
Option 1	a_1	a_2	a_3	...	a_n
Option 2	β_1	β_2	β_3	...	β_n
Option 3	γ_1	γ_2	γ_3	...	γ_n
...				...	

Credence and Betting Odds

By proposing a bet on p we give the subject a possible course of action from which so much extra good will result to him if p is true and so much extra bad if p is false. Supposing the bet to be in goods and bads instead of in money, he will take a bet at any better odds than those corresponding to his state of belief; in fact his state of belief is measured by the odds he will just take

Credence and Betting Odds

	p	$\neg p$
Bet(α, β)	α	β
Leave(γ)	γ	γ

$$\text{Cr}_X(p) =_{\text{df}} \inf \left\{ \frac{U(\gamma) - U(\beta)}{U(\alpha) - U(\beta)} : X \text{ would choose } \textbf{Bet}(\alpha, \beta) \text{ over } \textbf{Leave}(\gamma) \right\}$$

Credence and Betting Odds

Ramsey notes credences are identical to betting odds on his approach, but emphatically **does not *define* credences as betting odds**. Literal bets are only one way to measure an subject's credences, which is in practice complicated by the pleasure people take in, or the aversion the feel towards, taking bets. For Ramsey *any* choice makes a data point in measuring credences (as illustrated by the crossroads example).

I. Reducing Credence to Utility

II. Measuring Utility

Four-Step Plan

1. Rank the outcomes

2. Identify an *ethically neutral* proposition with credence ½

3. Determine the Utilities

4. Determine the Credences

1. Rank the Outcomes

If then we had the power of the Almighty, and could persuade our subject of our power, we could, by offering him options, discover how he placed in order of merit all possible courses of the world.

1. Rank the Outcomes

- Ramsey uses Greek letters α, β, for what he calls "possible worlds", but really they are better thought of as being *total* outcomes, i.e. specifications of all states of affairs which the agent cares about.

- Ramsey assumes:

   A. That these total outcomes are totally ordered by the preference relation $\leq$.

   B. That there are some outcomes between which the agent is *not* indifferent: i.e. $\alpha < \beta$.

1. Rank the Outcomes

If then we had the power of the Almighty, and could persuade our subject of our power, we could, by offering him options, discover how he placed in order of merit all possible courses of the world.

2. Identify an Ethically Neutral Proposition with Credence ½

- An *ethically neutral* proposition is a state of affairs such that the agent is completely indifferent as to whether or not it is true.

- If p is ethically neutral, both p and its negation $\neg p$ are compatible with every maximal outcome α, and the agent is indifferent between $\alpha \wedge p$ and $\alpha \wedge \neg p$.

2. Identify an Ethically Neutral Proposition with Credence ½

The agent has **credence ½** in an ethically neutral proposition p if and only if there are some outcomes $\alpha < \beta$ such that the agent is indifferent between the following options:

	p	$\neg p$
Option 1	α	β
Option 2	β	α

3. Determine the Utilities

The *value difference* between α and β equals the value difference between γ and δ, written **$\alpha\beta = \gamma\delta$**, if and only if, for some ethically neutral p with credence ½, the agent is indifferent between the following options:

	p	$\neg p$
Option X	α	δ
Option Y	β	γ

3. Determine the Utilities

- At this point Ramsey introduces a set of axioms to guarantee these definitions are well behaved. These axioms are constraints on the agents preferences that characterise what Ramsey calls *coherent* behaviour.

- Now pick an arbitrary α and β such that $\alpha < \beta$. Then set $U(\alpha) = 0$, $U(\beta) = 1$.

- With the above definition of value difference, this uniquely specifies a value/utility function U from maximal outcomes to real numbers.

4. Determine the Credences

	p	$\neg p$
Bet(α, β)	α	β
Leave(γ)	γ	γ

$$Cr_X(p) =_{df} \inf \left\{ \frac{U(\gamma) - U(\beta)}{U(\alpha) - U(\beta)} : X \text{ would choose } \textbf{Bet}(\alpha, \beta) \text{ over } \textbf{Leave}(\gamma) \right\}$$

In addition, Ramsey defines *conditional credence* and shows that the function Cr thus defined must be a **probability function.**

Ramsey's Representation Theorem

If an agent *X* behaves coherently, then all the choices that agent makes will maximise expected utility with respect to some uniquely determined probability function Cr_X, and a real-valued utility function *U* that is unique up to choice of zero and unity.

Why It Matters

- Ramsey's Theorem gives a precise meaning to the concept of *credence*.

- It gives us a way of understanding *why* credences should obey the laws of probability. "The laws of probability are the laws of coherence."

- It also gives a precise meaning to the concept of a *utility*.

- It suggests an account of our ability to interpret the behaviour of other people to make inferences about their beliefs and desires.

Idealisation

- Real-life agents are not truly coherent, and different measurements of their credences may yield different results.

- The result "cannot be established without a certain amount of hypothesis or fiction."

- Analogy with Newtonian time intervals.

Other Representation Theorems

1. Bruno DeFinetti

2. Von Neumann and Morgenstern

3. L.J. Savage

Prep for Tuesday 31st

1. On the message board, post a sentence or short passage in Ramsey's paper that you find interesting but hard to understand. Explain as best you can what you find puzzling about the sentence/passage in question.

2. Respond to one other person's message. Your response can be your interpretation of the passage, an answer to the original poster's question, or an additional question about the same passage.