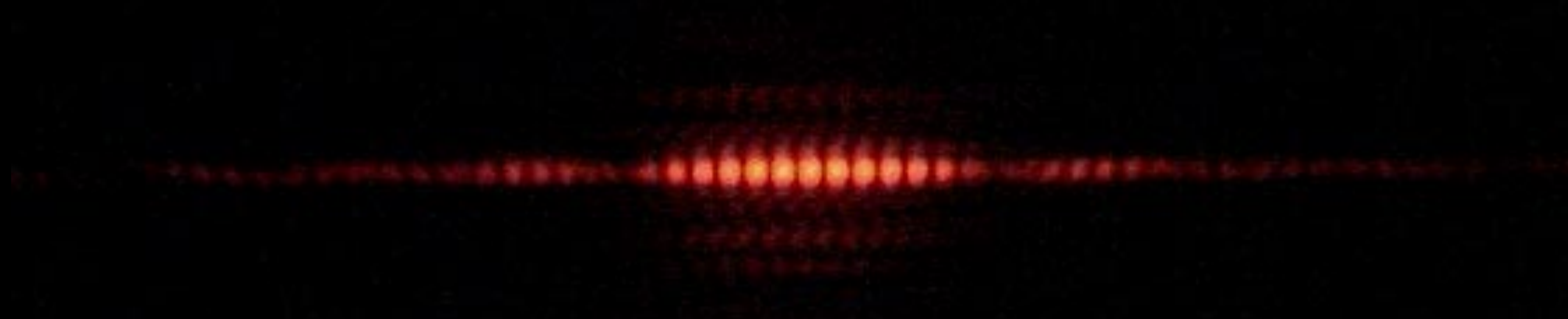
Crash Course in Quantum Mechanics

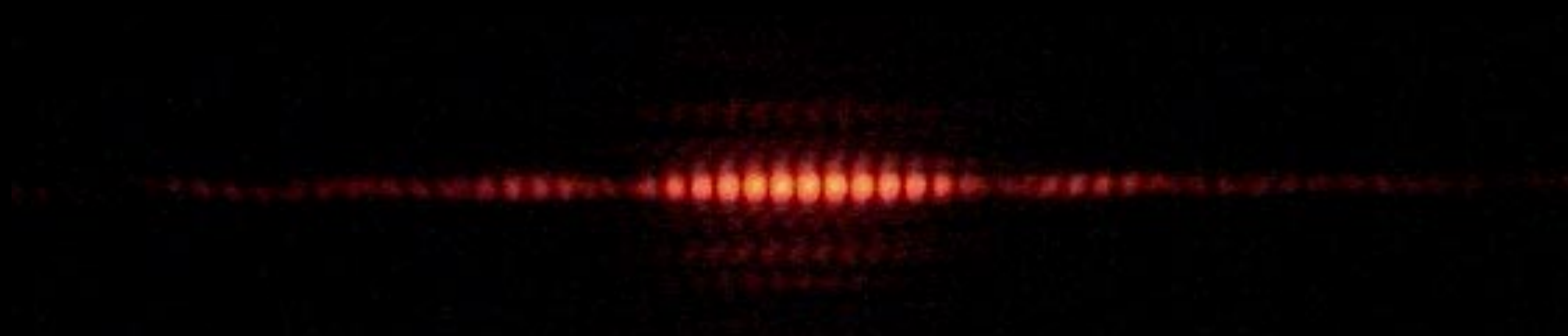


Dan Hoek — PHI 371 Foundations of Probability and Decision Theory — Princeton — April 2020

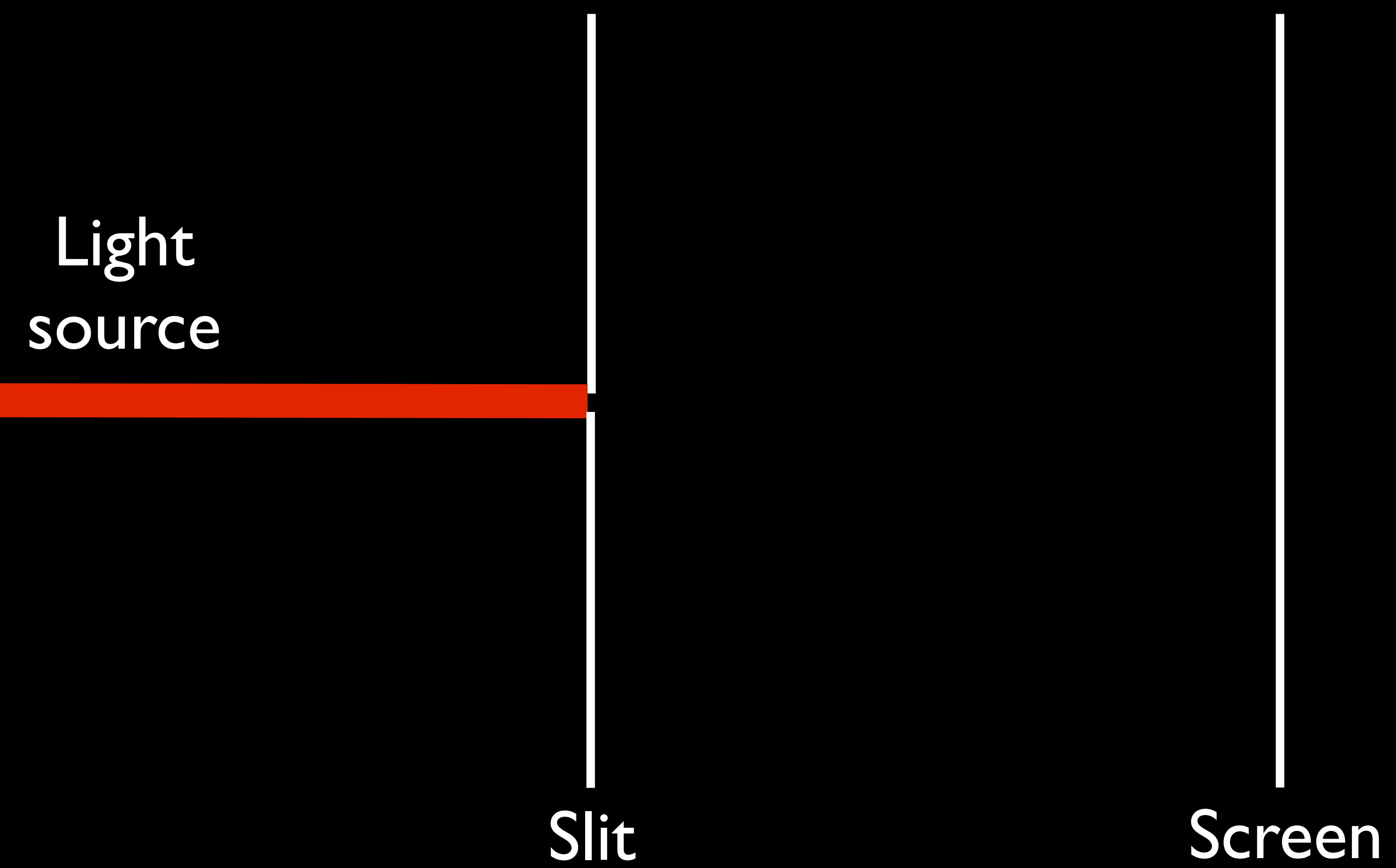
I. The Double Slit Experiment

II. The Quantum Recipe

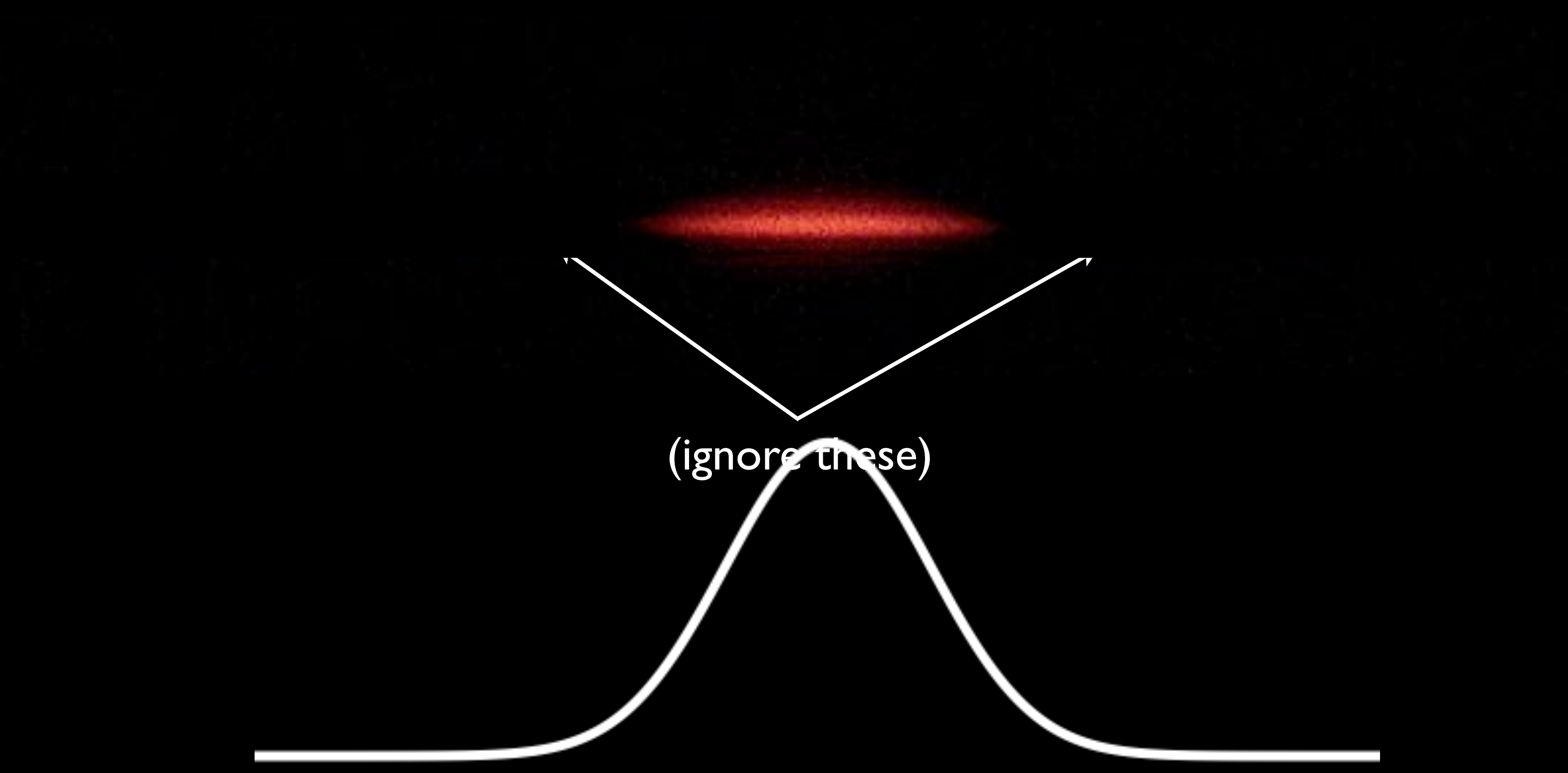
III. Many Worlds



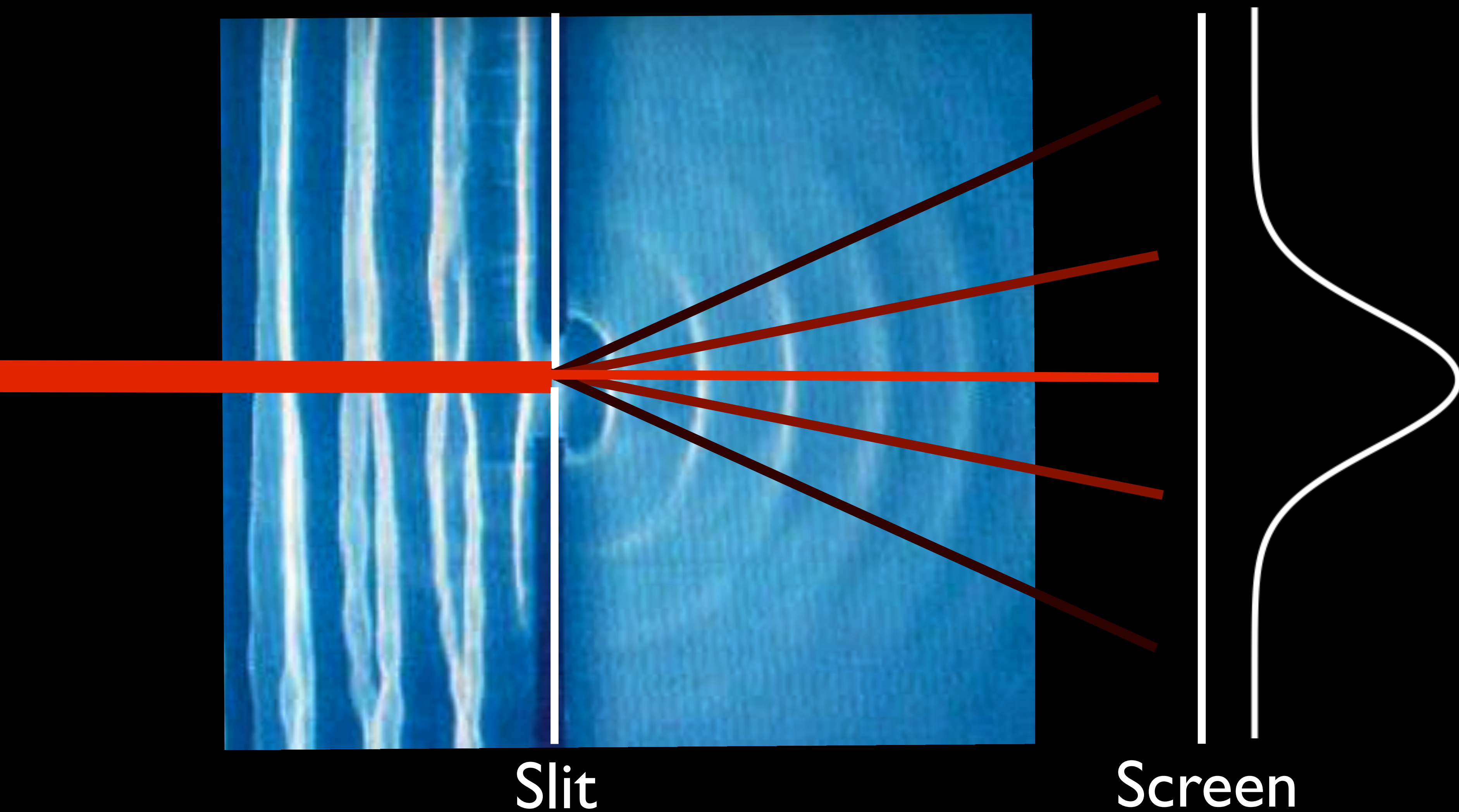
Single slit experiment



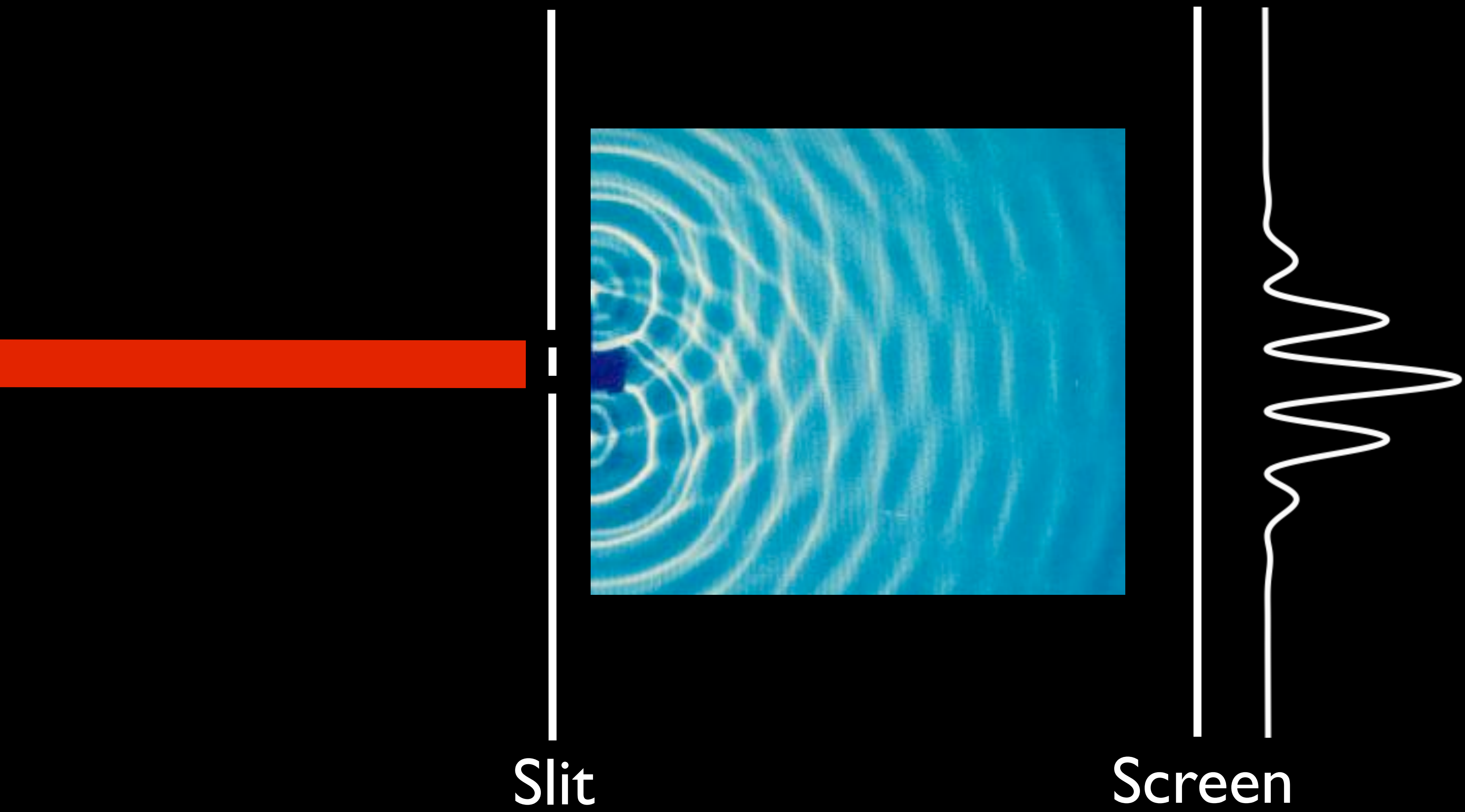
Single slit pattern



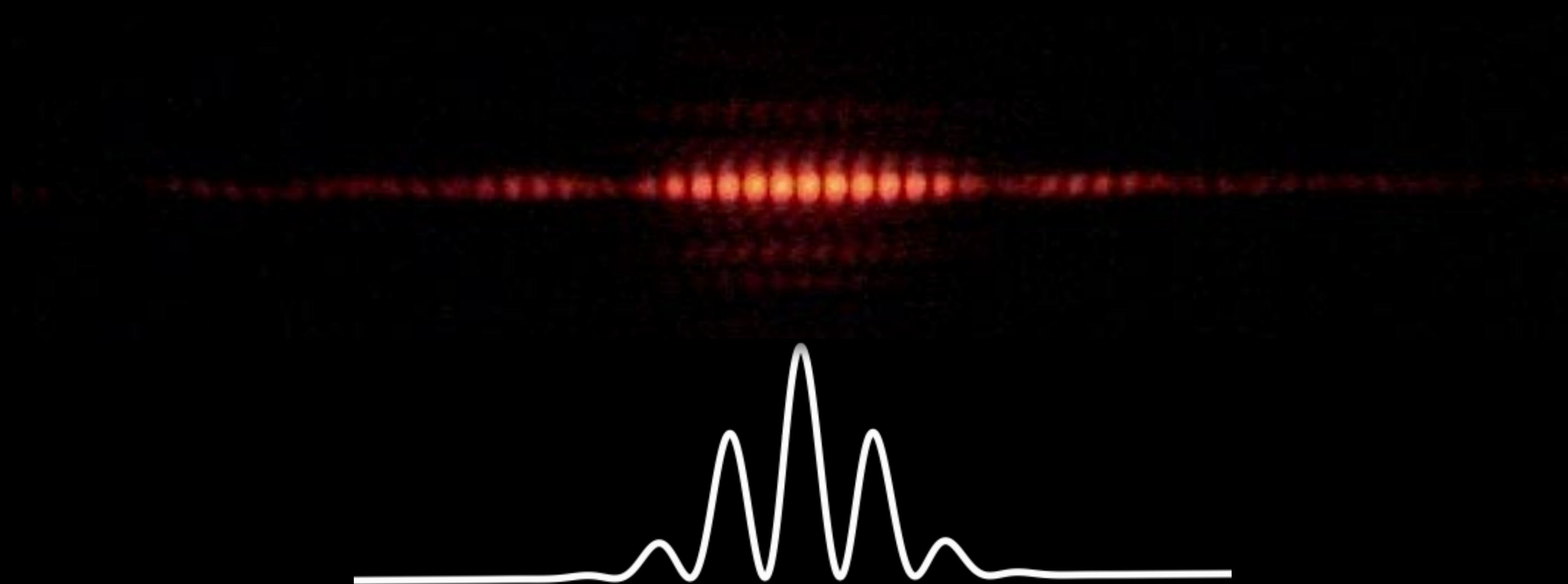
Single slit experiment



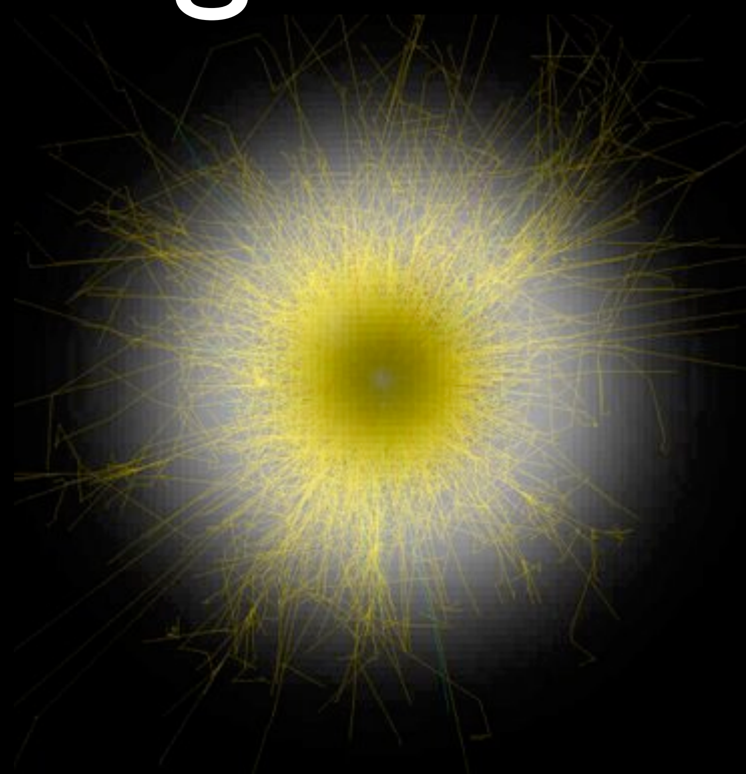
Double slit experiment



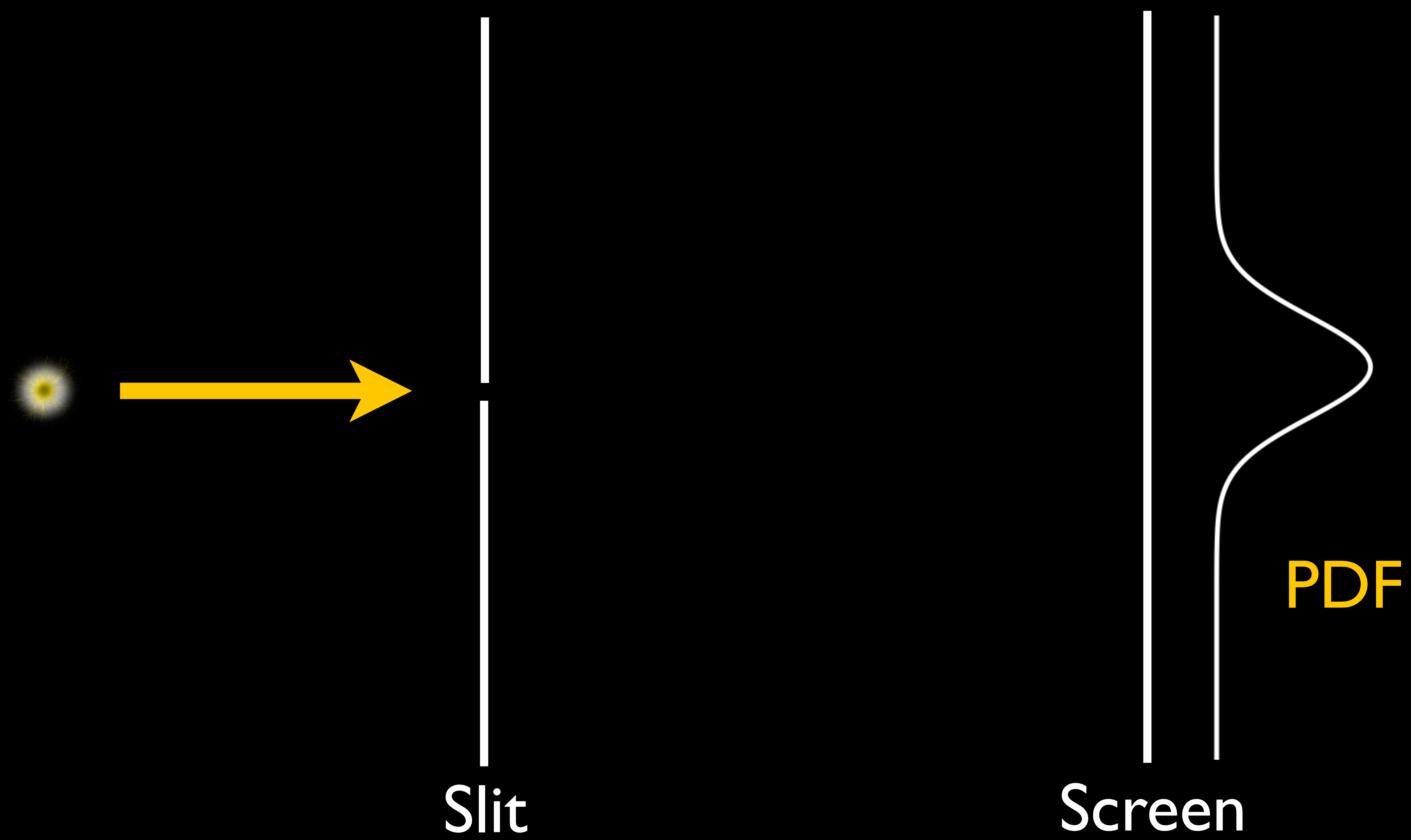
Double slit pattern



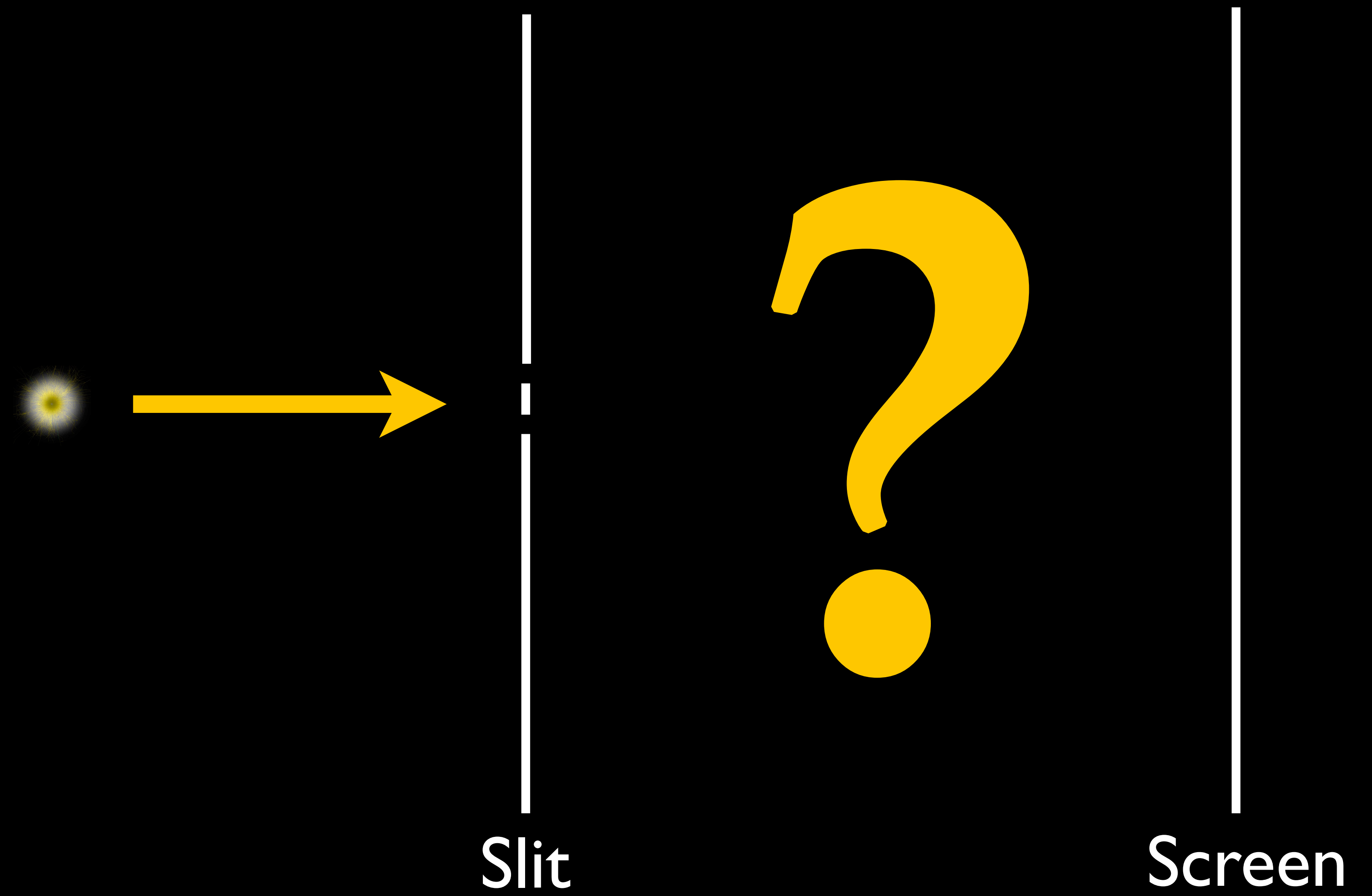
What if we just let one photon at a time through the slits?



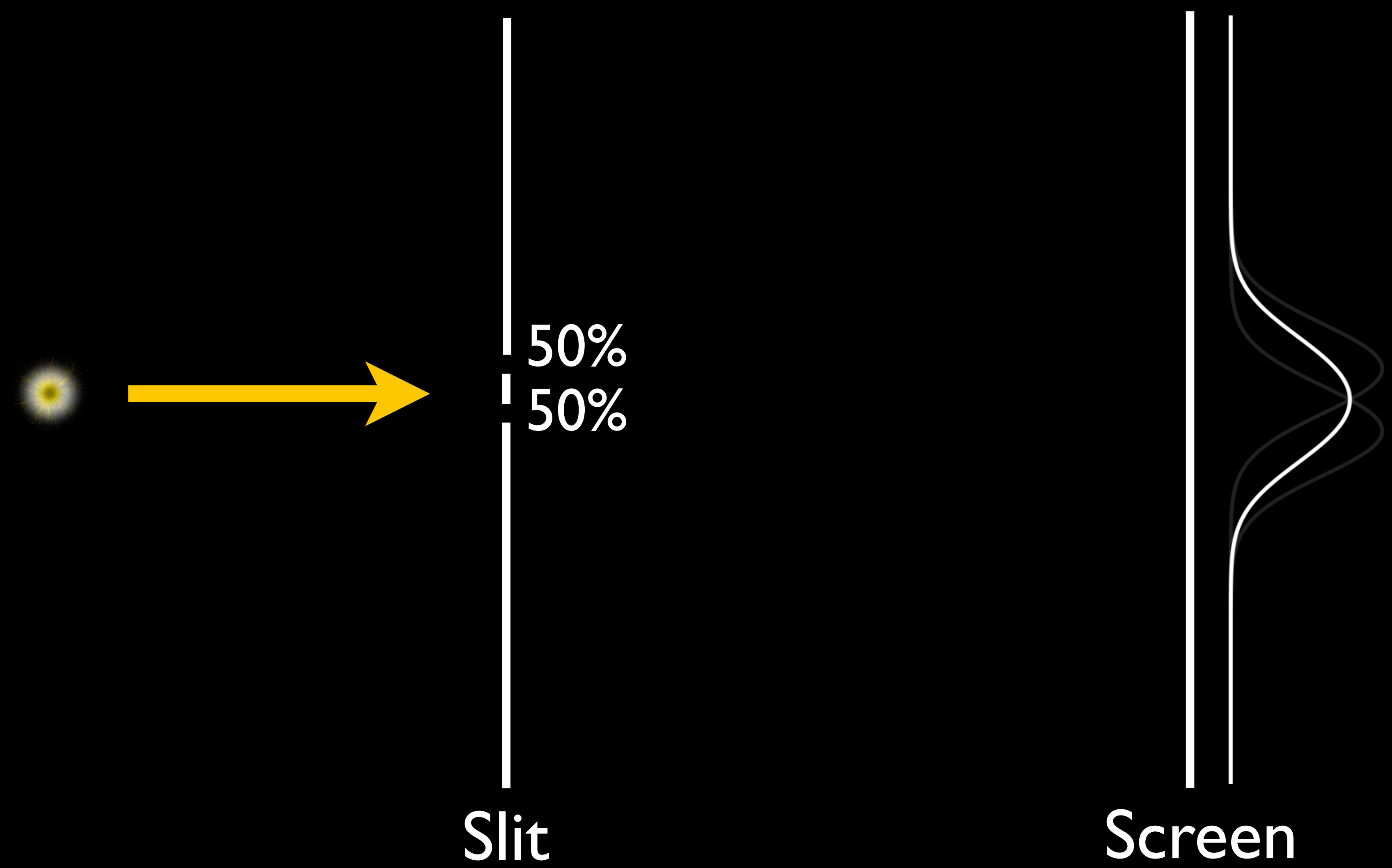
Single slit experiment



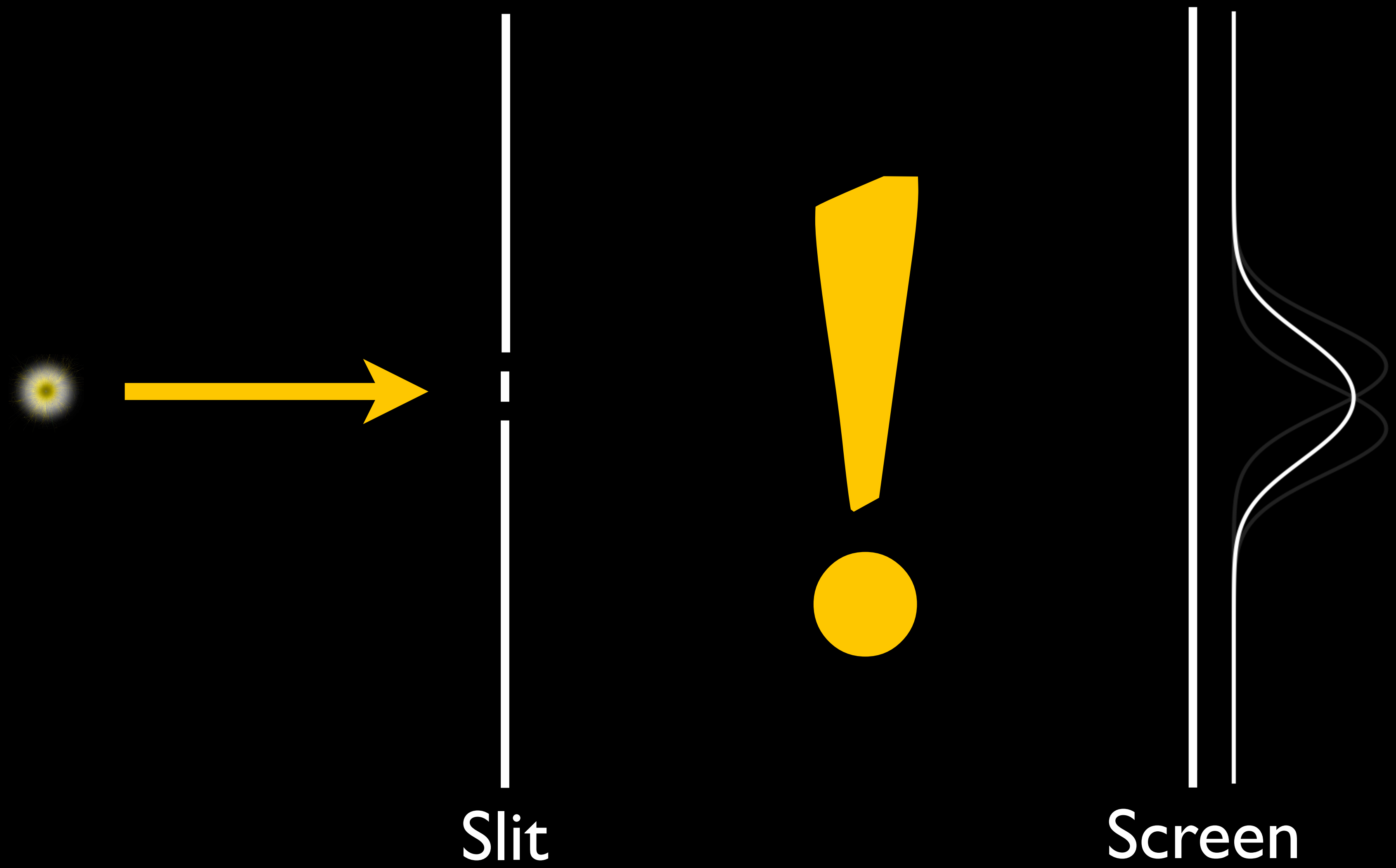
Double slit experiment



Double slit experiment

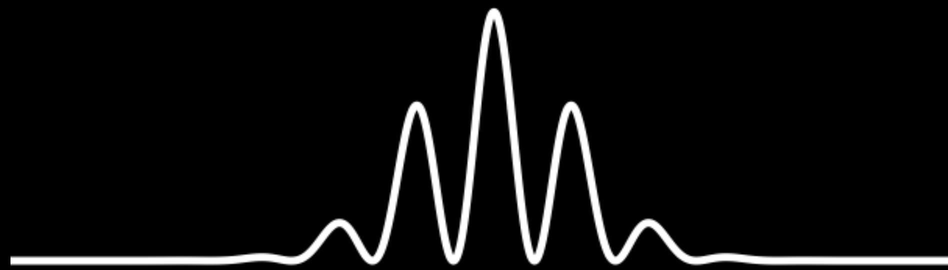


Double slit experiment

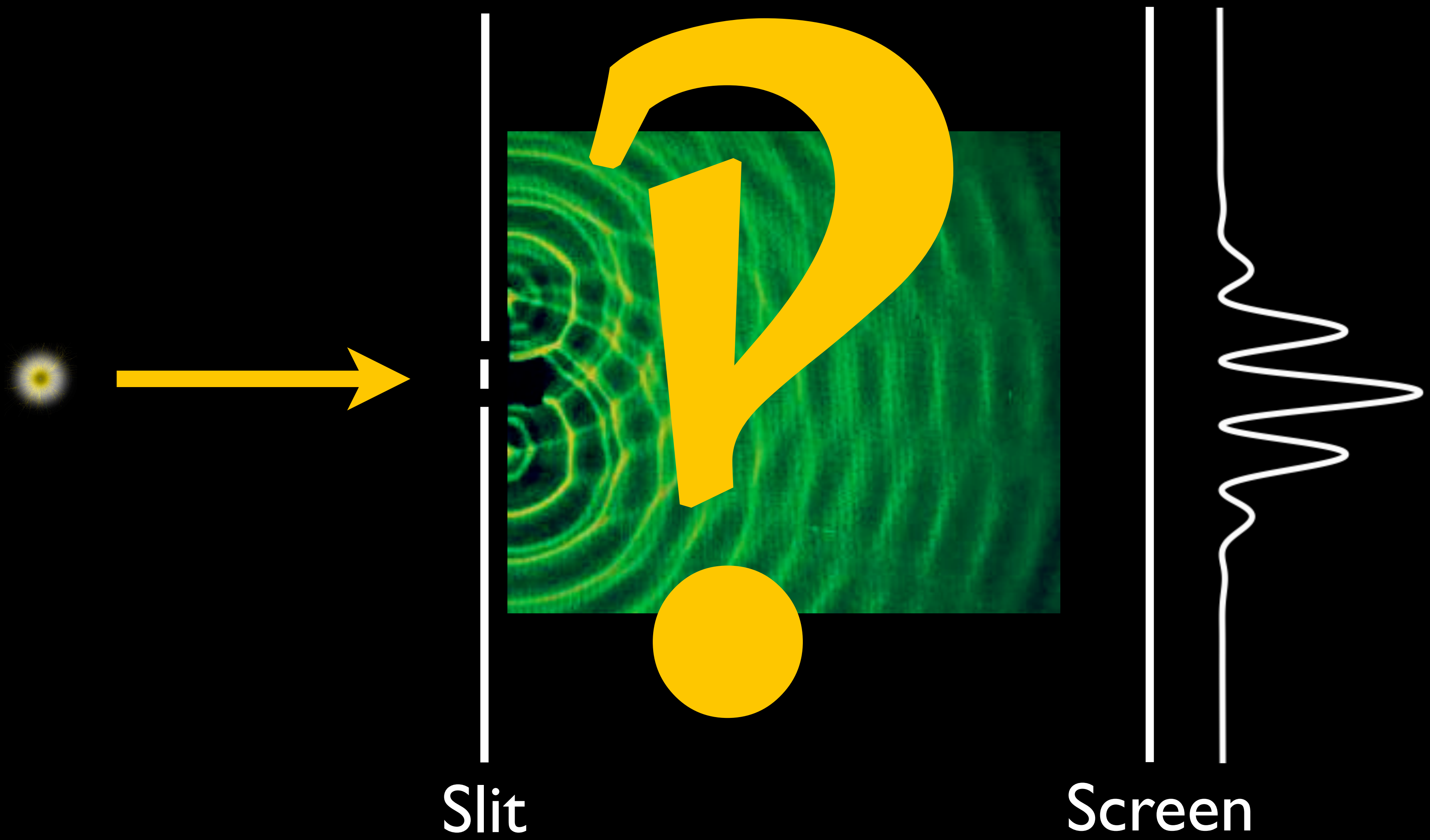


Hypothesis

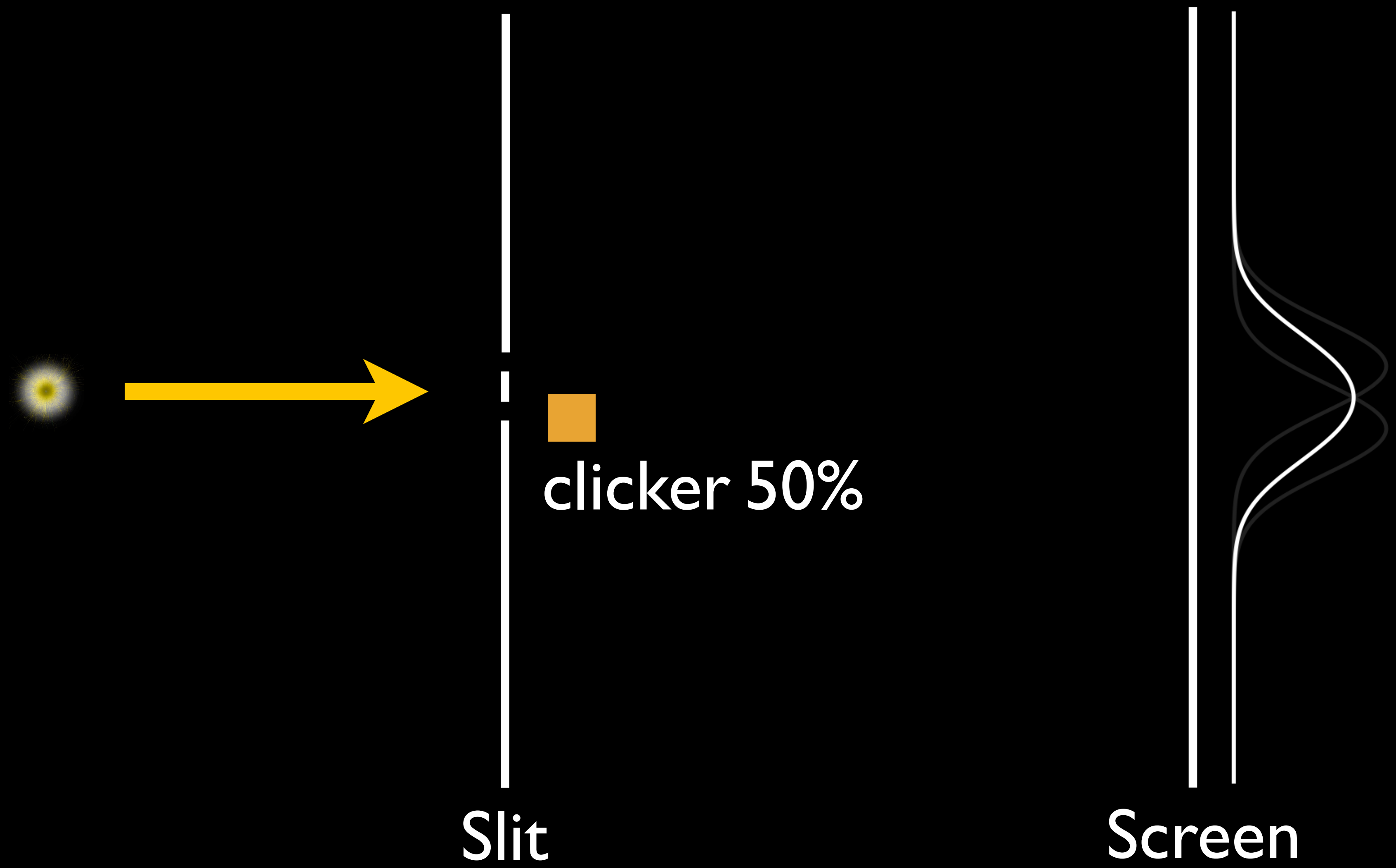
Result



Double slit experiment



Modified experiment

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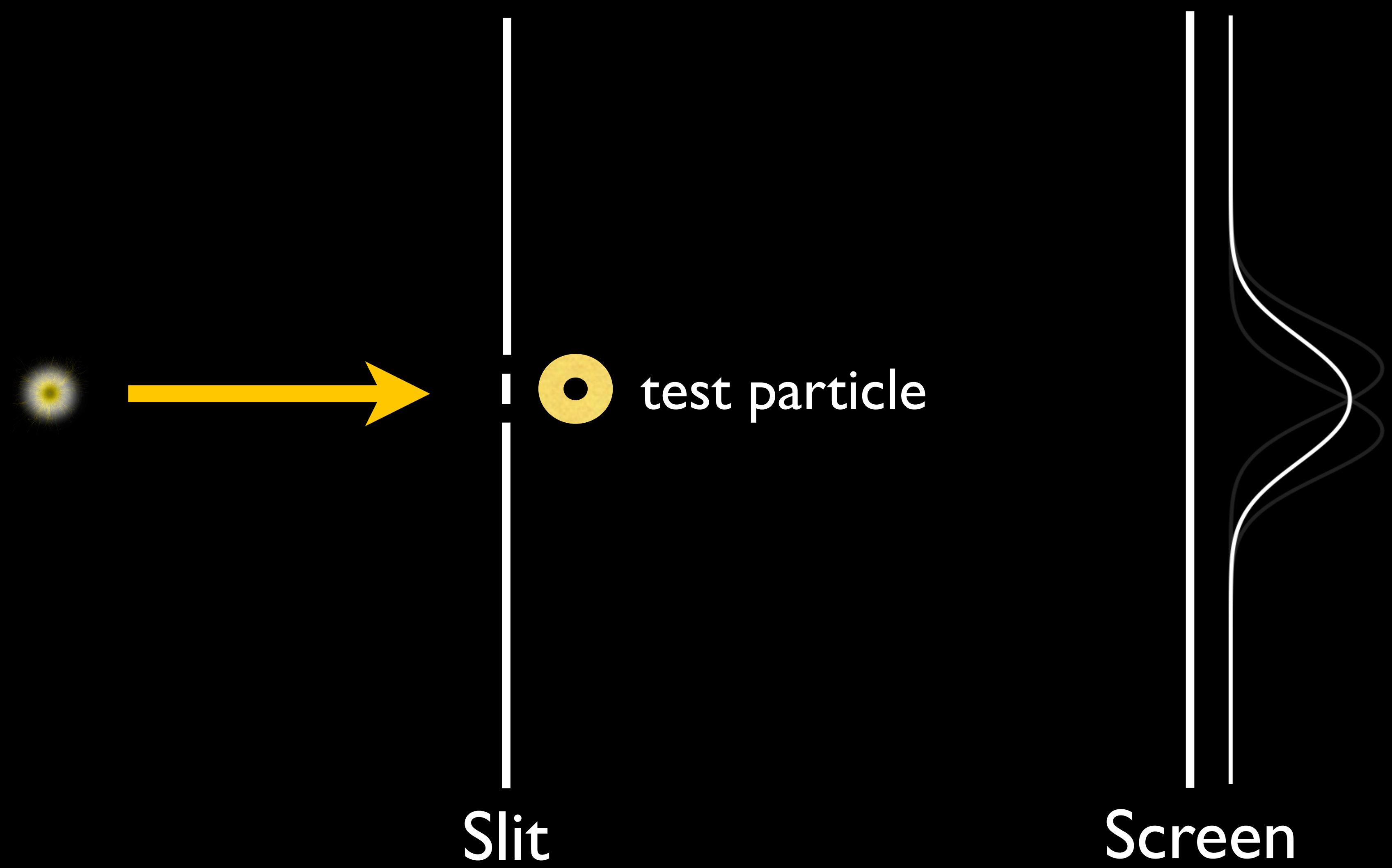
Modified experiment

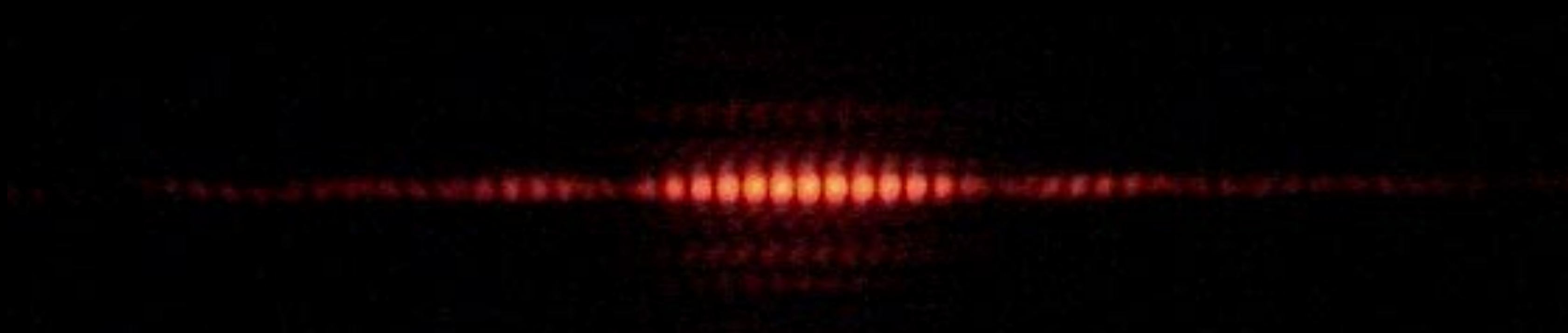


?

I. The Double Slit Experiment

II. The Quantum Recipe

III. Many Worlds



Here is a partial list of the elementary principles of Quantum Theory:[1]

(I) Any isolated quantum mechanical system is characterized by a *state function* $\$_S(t)$.

(II) So long as S remains isolated, $\$_S(t)$ evolves deterministically in accordance with the Schrodinger equation (or in accordance with one of the relativistic generalizations of the Schrödinger equation).

(III) For any complete compatible set of observables O of S, $\$_S$ can always be expressed as a sum or a "superposition" of eigenstates of O, as follows:

(1) $\$_S = c_1 O_1 + c_2 O_2 + \cdots$

where the c_i are complex numbers, and the O_i represent quantum states (eigenstates of O) of S in which O has the particular value o_i such that if $i \neq j$ then $o_i \neq o_j$.

(IV) When a measurement of O is carried out on S in state $\$_S$ the probability of obtaining $O = o_k$ is equal to the absolute square of the amplitude c_k of O_k in the state function $\$$.

(V) When a measurement of O is carried out with the result that $O = o_k$ then the state of S "collapses," or is "reduced" instantaneously into the eigenstate O_k.

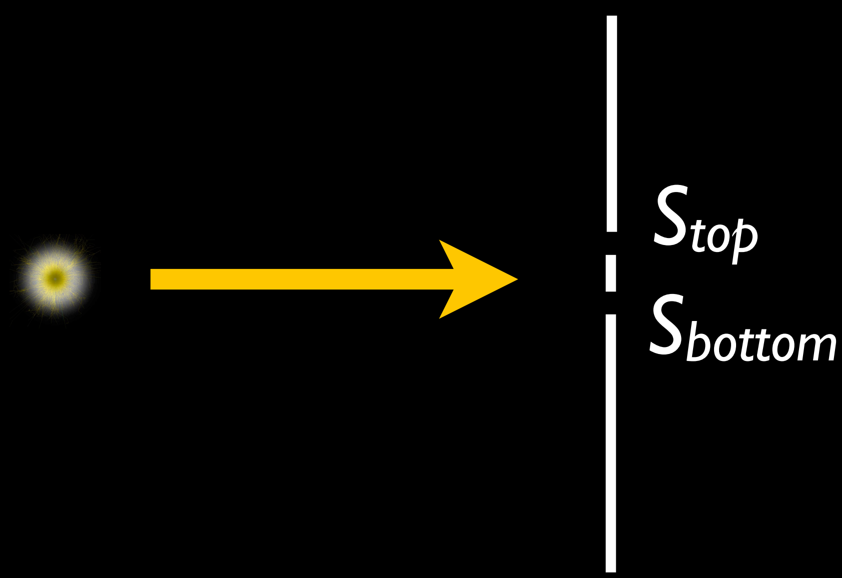
Observables

Informally, observables are *Physical properties of a system that you can measure*.

- What is the position of the particles?
- What is their velocity / momentum?
- What is their charge?
- Which slit did the particle go through?
- What is the spin of the particle?

Eigenstates

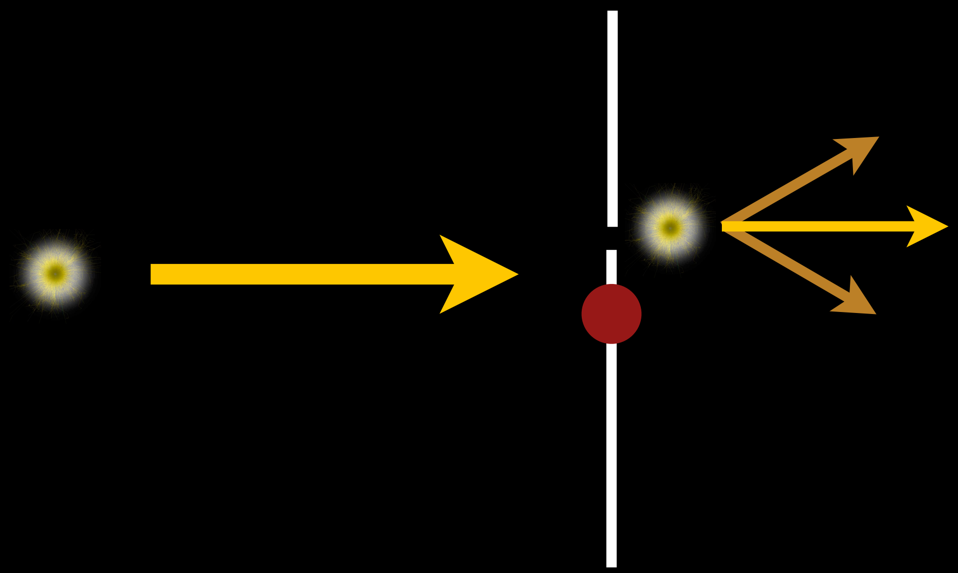
The Eigenstates of an Observable correspond to the *particular, specific values the quantity might take*. For our electron, the observable *S*, *Which slit?* has two eigenstates: S_{top} and S_{bottom}



Eigenstates

For instance, if the bottom slit is blocked and the particle still goes through the slits, it is described as being in an Eigenstate w.r.t. *S*:

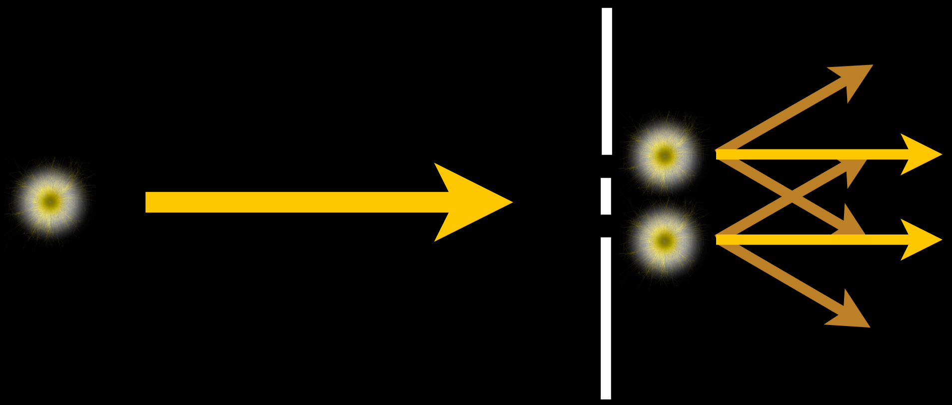
$$\psi_{\text{particle}} = S_{\text{top}}$$



Superposition

Besides being in an eigenstate, a system can also be described as being in a *superposition of eigenstates* with respect to a given observable:

$$\psi_{particle} = a \cdot S_{top} + b \cdot S_{bottom}$$



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amplitudes

$$|a|^2 + |b|^2 = 1$$

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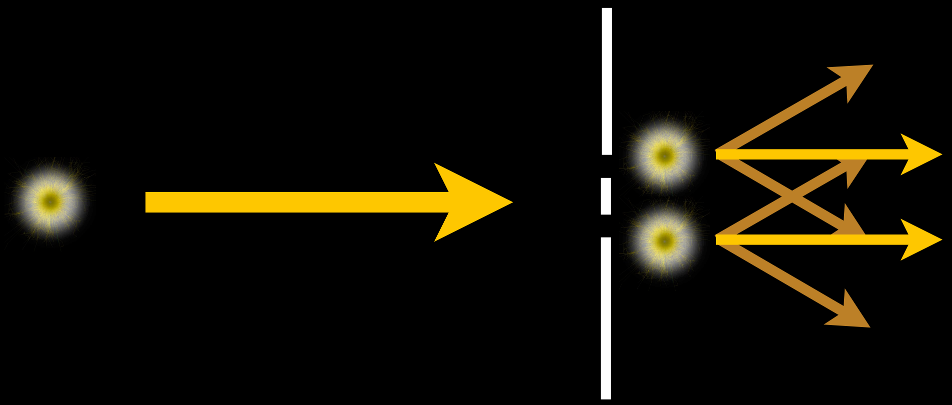
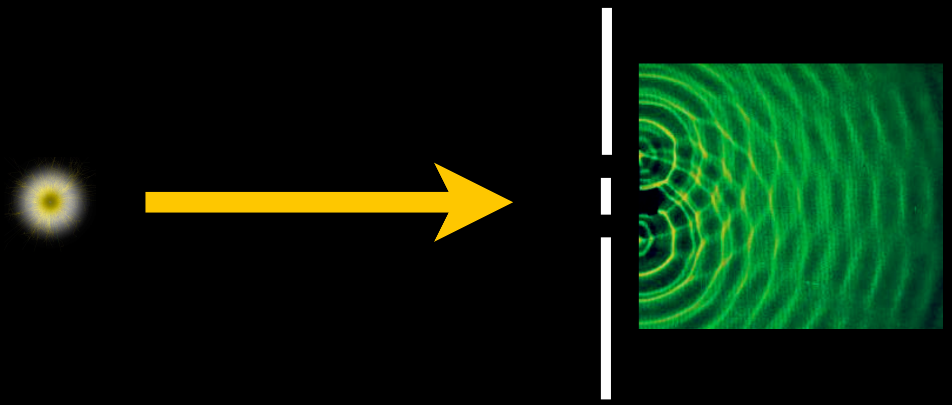


Schrödinger Equation

The Schrödinger equation describes how the state of a system *evolves over time*:

$$\psi_{\text{particle}}(t + \delta) = \frac{-i}{\hbar} \hat{H} \, \psi_{\text{particle}}(t)$$



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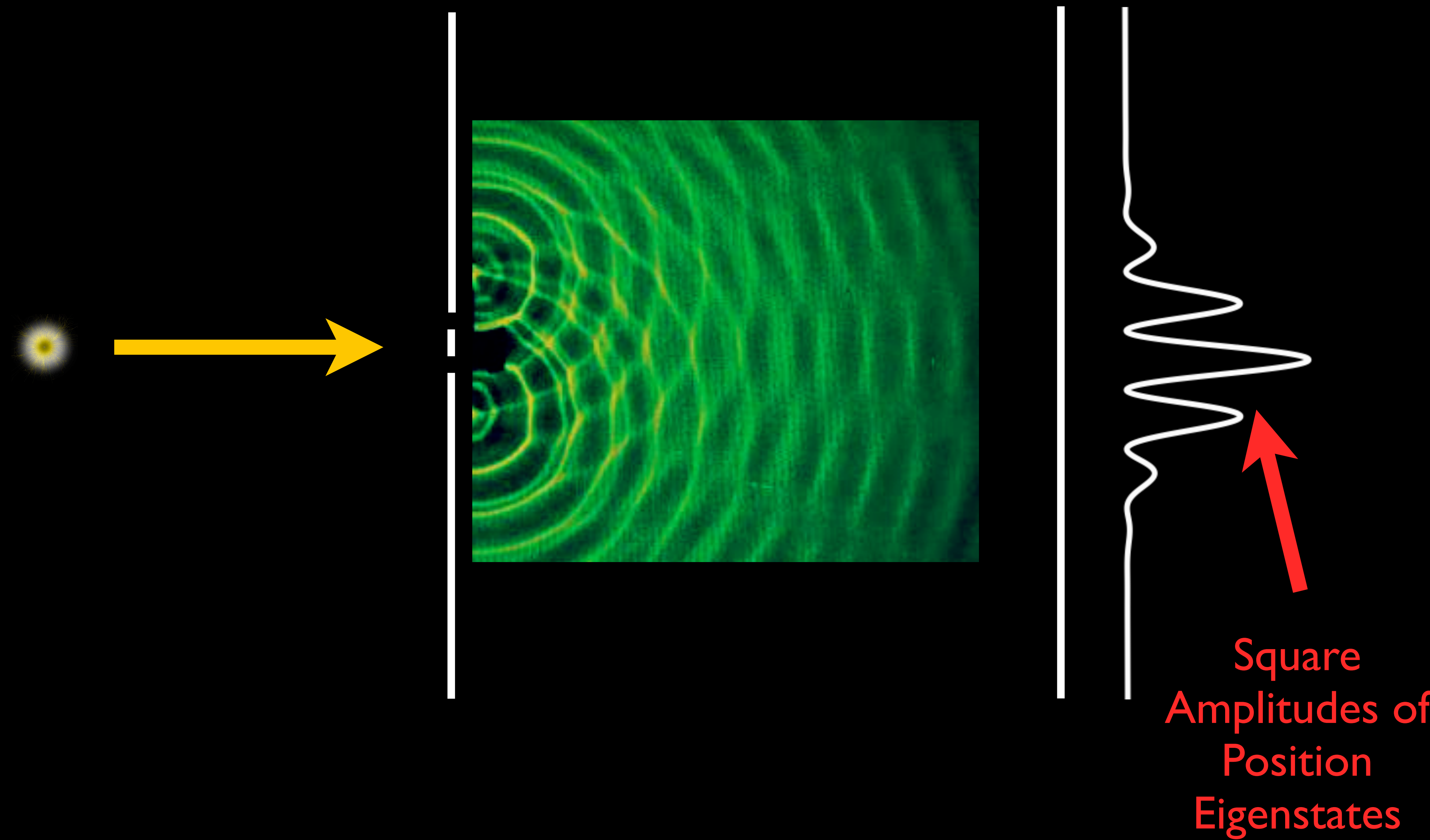
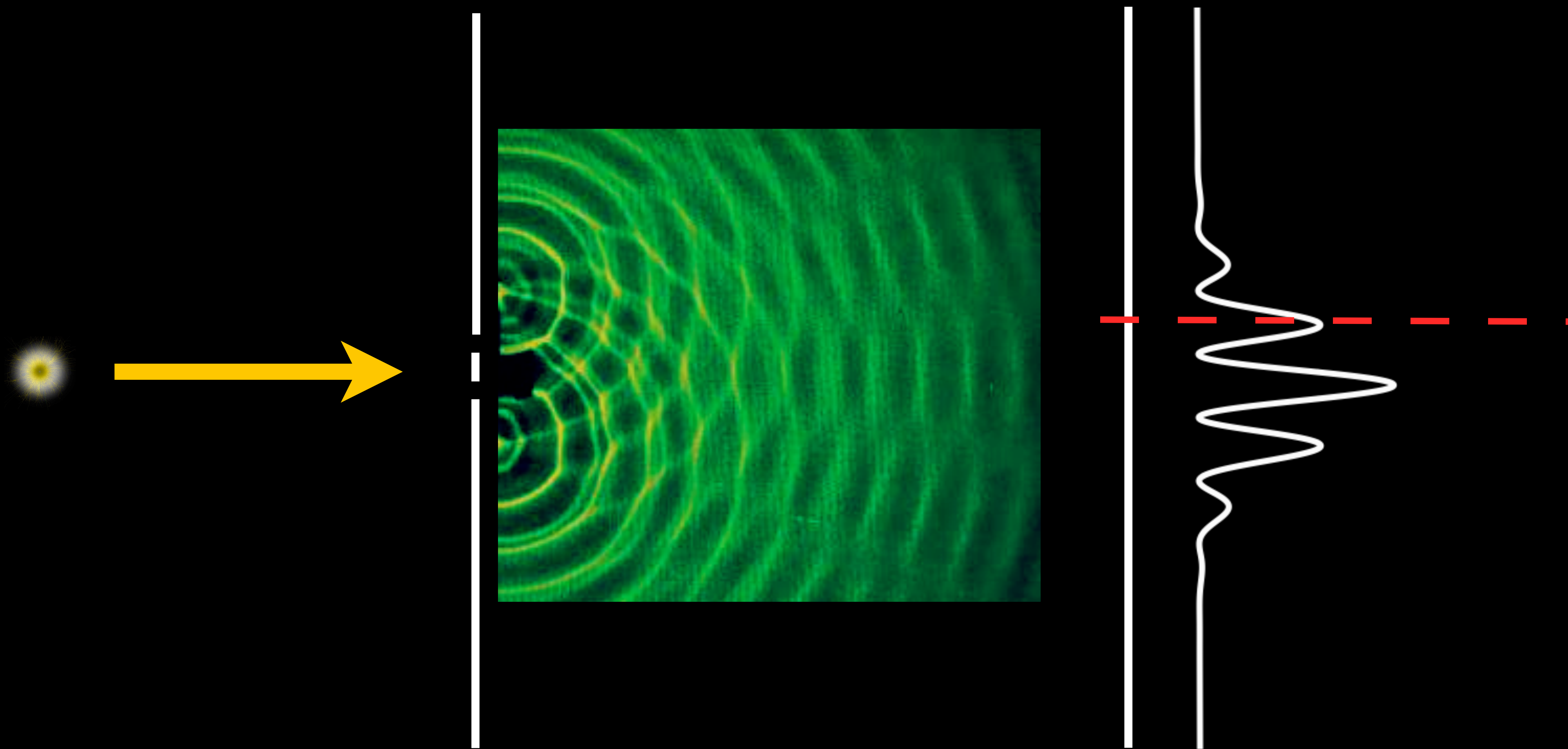
Schrödinger Equation



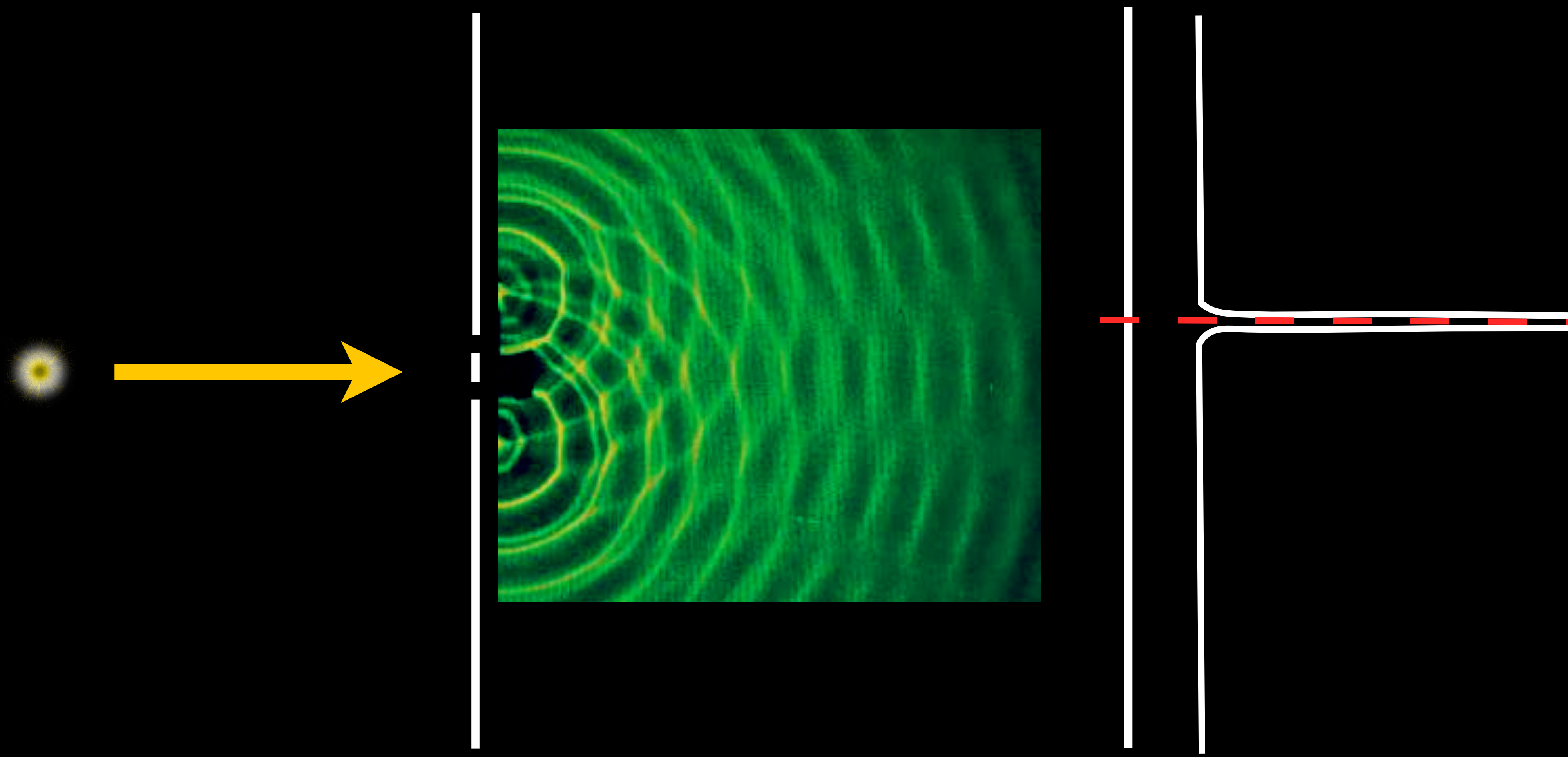
Collapse Postulate

Whenever a measurement of the system occurs, the state of the system *instantly collapses into one of the eigenstates of the relevant observable*.

Collapse Postulate



Collapse Postulate



Collapse Postulate

Whenever a measurement of the system occurs, the state of the system *instantly collapses into one of the eigenstate of the relevant observable*.

The chance that it collapses into a given eigenstate is given by the corresponding square amplitude in the state prior to collapse (Born Rule)

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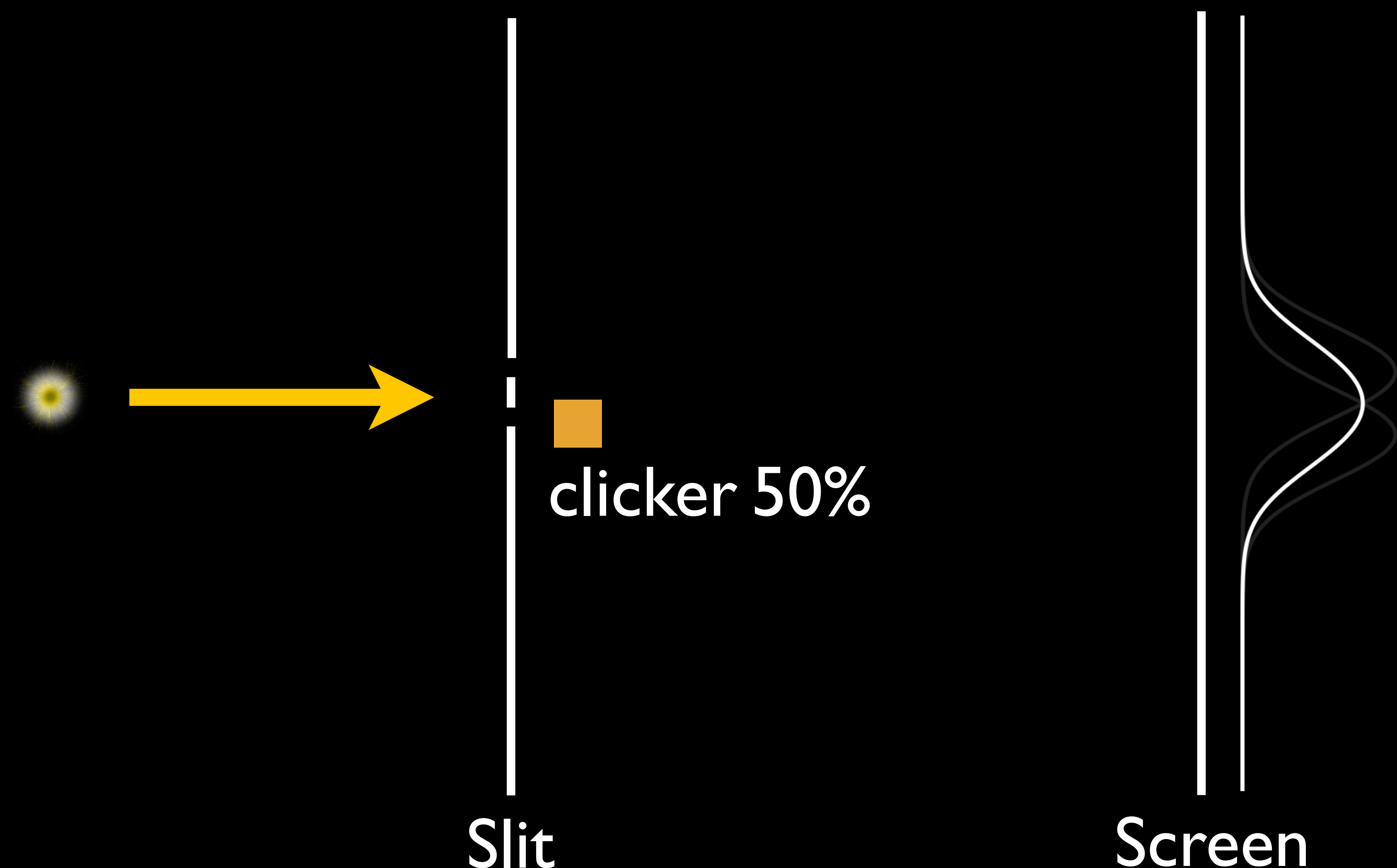
Collapse Postulate



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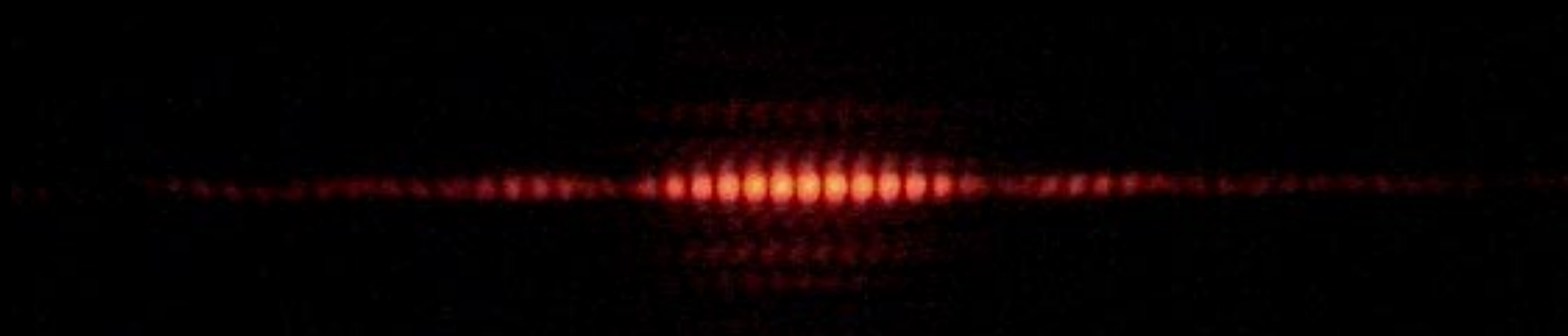
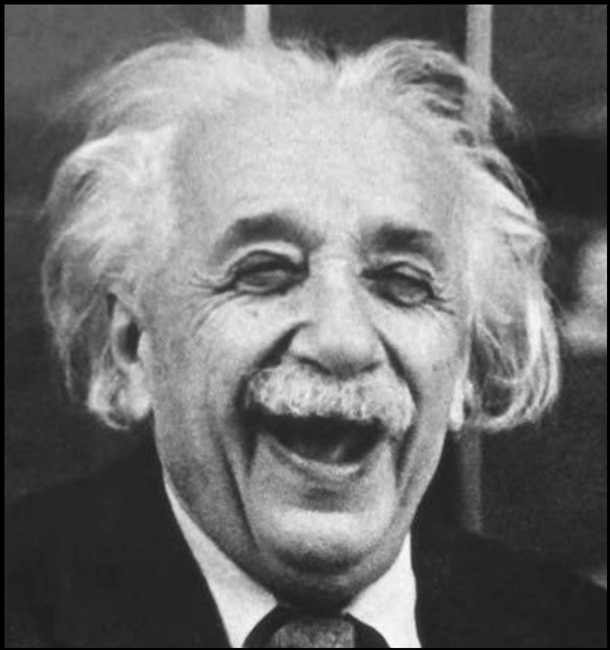


The Measurement Problem



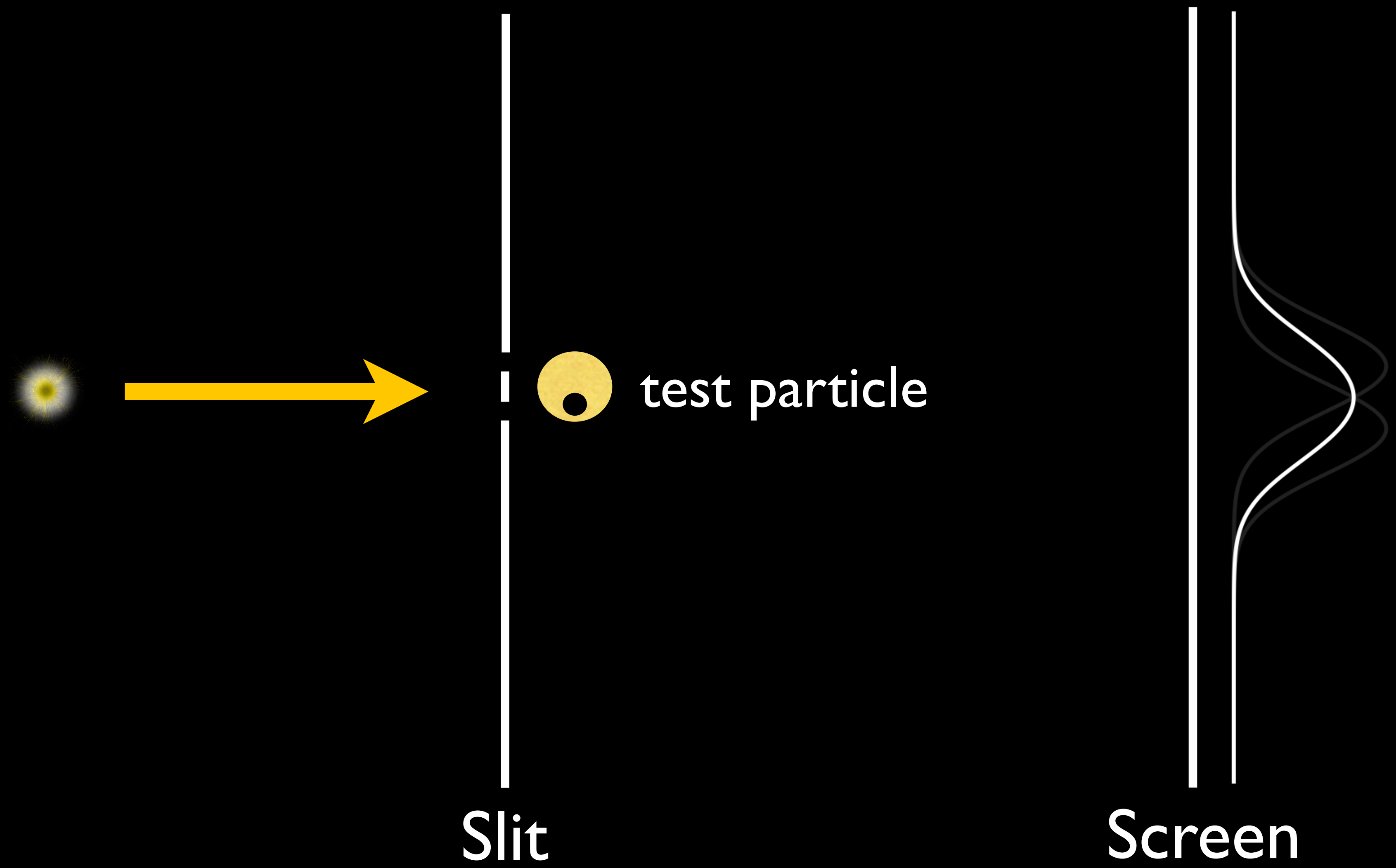
The Measurement Problem

Difficulties with the Collapse Postulate:

- Why does the fundamental dynamic law of QM, the Schrödinger Equation, have *exceptions*?
- Under what conditions do those exceptions take place? What even is a measurement?
- How is can the wave to collapse everywhere simultaneously?

What happens if we drop the Collapse Postulate?

Modified experiment



Entanglement

$$\psi_{sp,\, tp}(t_0) \quad = \quad S_{initial} \otimes T_{initial}$$

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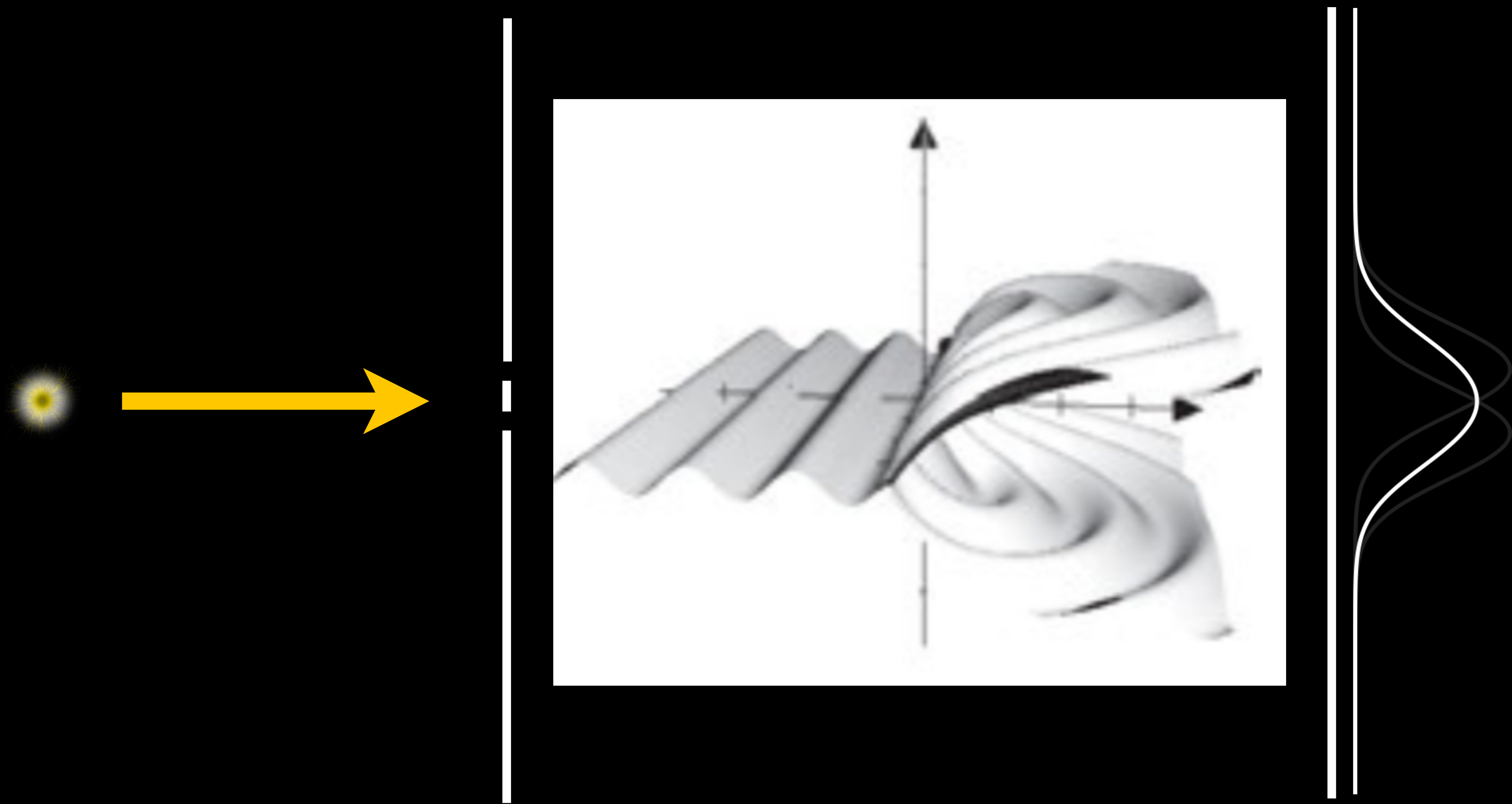
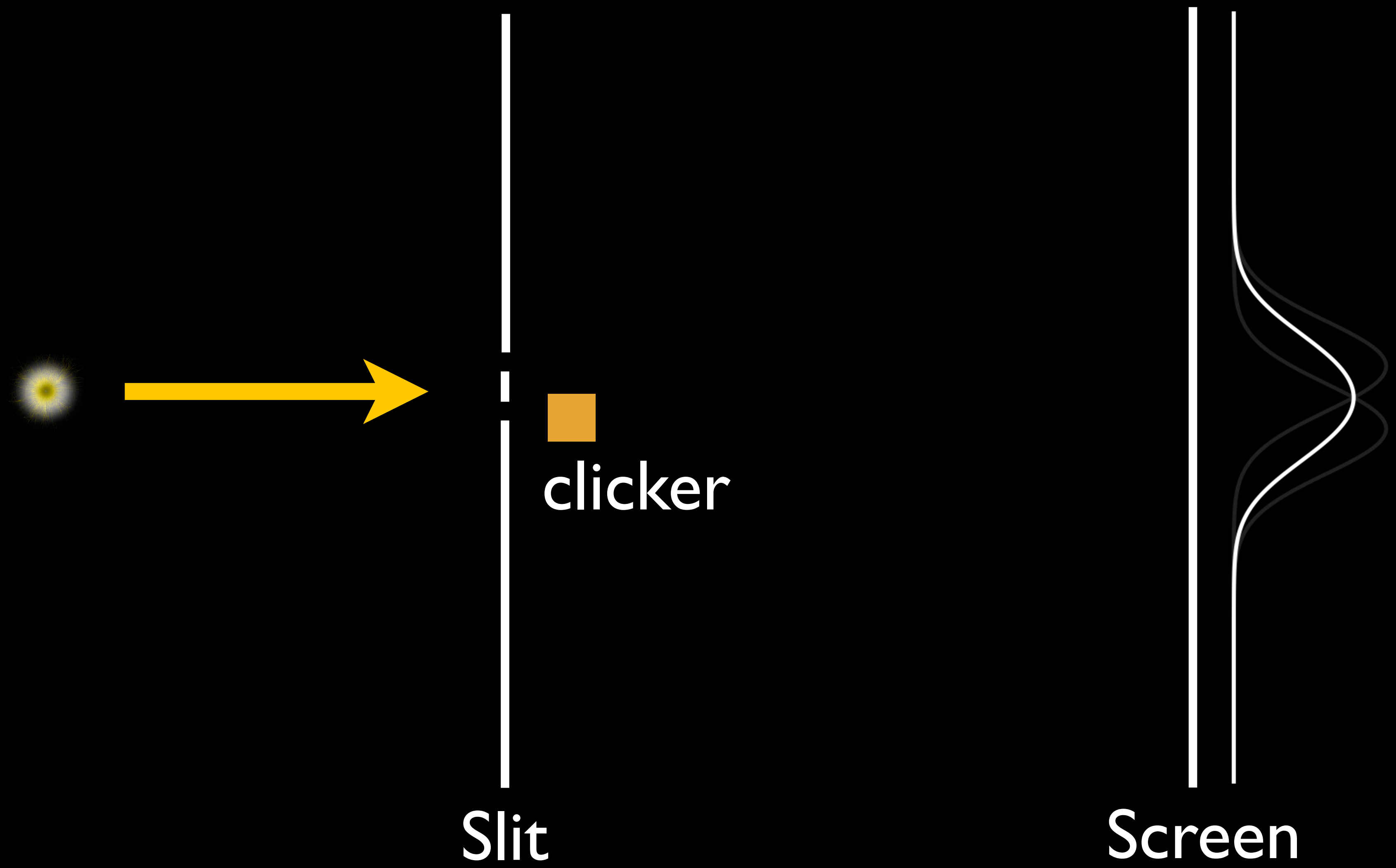
$$\psi_{\mathrm{sp,\,tp}}(t_0) \quad = \quad S_{\mathrm{initial}} \otimes T_{\mathrm{initial}}$$

$$\psi_{\mathrm{sp,\,tp}}(t_1) \quad = \quad a \cdot S_{\mathrm{top}} \otimes T_{\mathrm{top}} + b \cdot S_{\mathrm{bottom}} \otimes T_{\mathrm{bottom}}$$

Entanglement



Entanglement

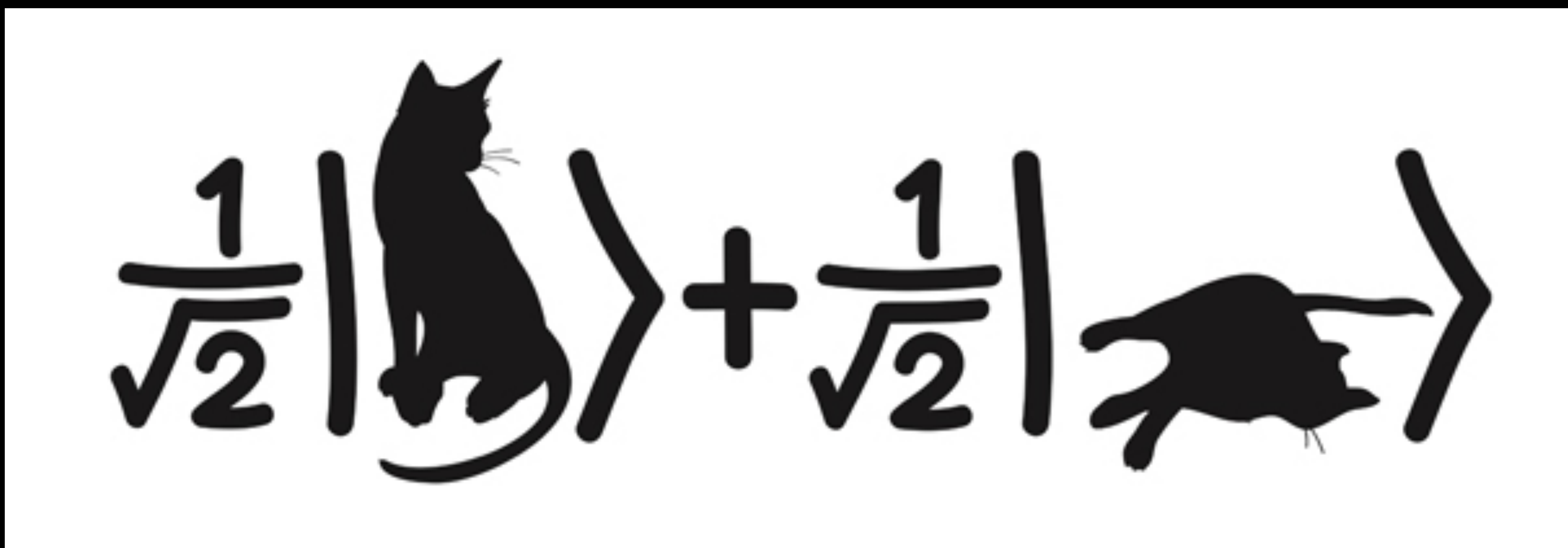


Entanglement

$$\psi_{\text{sp, cl}}(t_0) \quad = \quad S_{\text{initial}} \otimes C_{\text{initial}}$$

$$\psi_{\text{sp, cl}}(t_1) \quad = \quad a \cdot S_{\text{top}} \otimes C_{\text{silent}} + b \cdot S_{\text{bottom}} \otimes C_{\text{click}}$$

Schrödinger's Cat



Entanglement

$$\psi_{\text{sp, cl, exp}}(t_0) \quad = \quad S_{\text{initial}} \otimes C_{\text{initial}} \otimes E_{\text{initial}}$$

$$\begin{aligned}\psi_{\text{sp, cl}}(t_1) \quad = \quad & a \cdot S_{\text{top}} \otimes C_{\text{silent}} \otimes E_{\text{silent}} \\ & + \; b \cdot S_{\text{bottom}} \otimes C_{\text{click}} \otimes E_{\text{hears “click”}}\end{aligned}$$

Many Worlds

"The universe is constantly splitting into a stupendous number of branches, all resulting from the measurement like interactions between its myriads of components. Moreover, every quantum transition taking place on every star, in every galaxy, in every remote corner of the universe is splitting our local world on earth into myriads of copies of itself."

— DeWitt

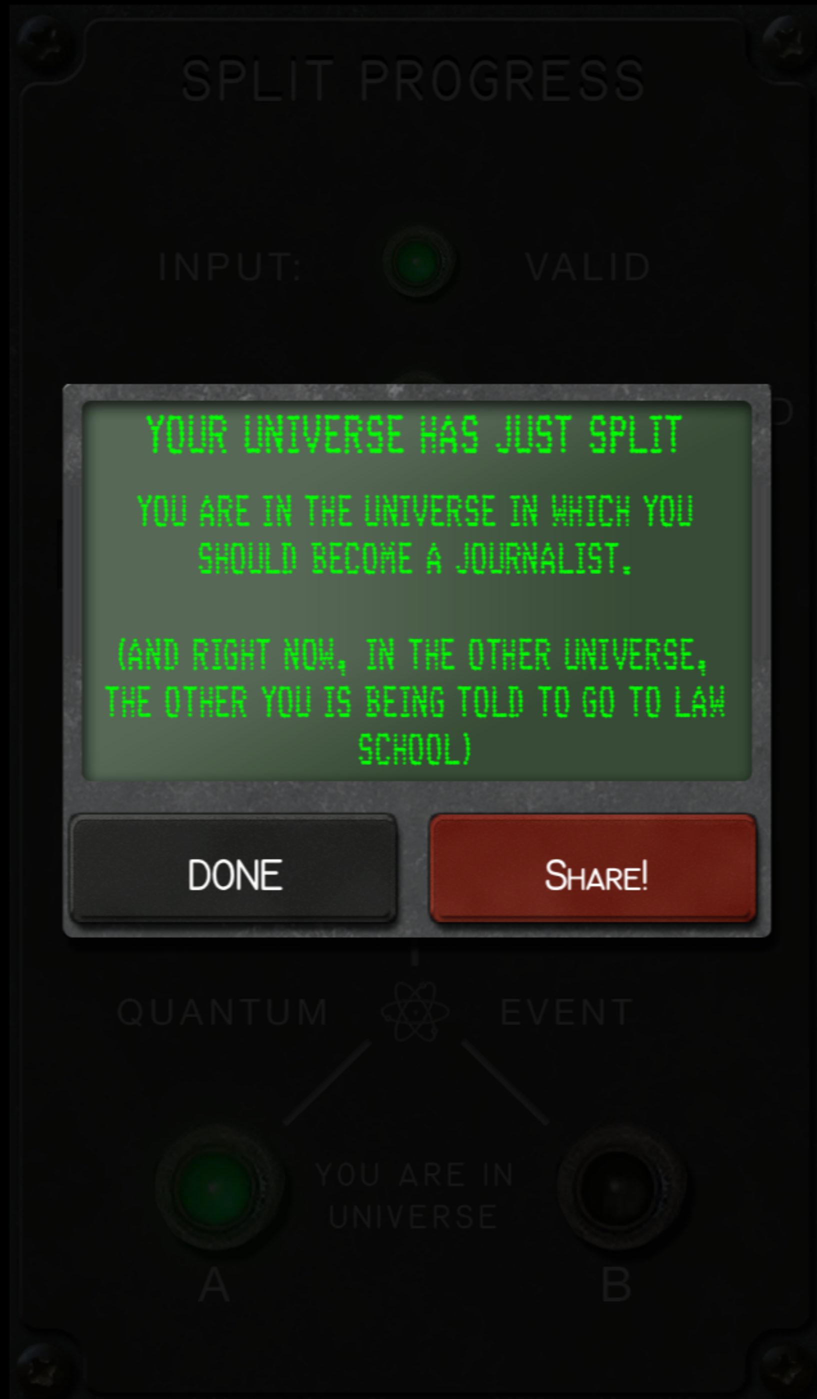
Many Worlds

Many Worlds

Universe Splitter


UNIVERSE
SPLITTER
QUANTUM-INDUCED
UNIVERSE BIFURCATION
IN ONE UNIVERSE, I WILL NOW:
Become a Journalist
IN THE OTHER ONE, I WILL NOW:
Go to Law School
SPLIT
UNIVERSE
SPLIT
CHART
0512
1024
2048
STATS
?
MANUAL


SPLIT PROGRESS
INPUT: VALID
INTERNET: CONTACTED
GENEVA ONLINE
DEVICE: READY
PHOTON: EMITTED
QUANTUM EVENT
YOU ARE IN UNIVERSE
A
B


SPLIT PROGRESS
INPUT:
VALID
YOUR UNIVERSE HAS JUST SPLIT
YOU ARE IN THE UNIVERSE IN WHICH YOU SHOULD BECOME A JOURNALIST.
(AND RIGHT NOW, IN THE OTHER UNIVERSE, THE OTHER YOU IS BEING TOLD TO GO TO LAW SCHOOL)
DONE
SHARE!
QUANTUM
EVENT
YOU ARE IN UNIVERSE
A
B